Department of Aerospace Engineering
AS3030 Vibrations
Aug – Dec 2010
Quiz 2 Solutions

1.

$$\begin{aligned}
\text{Mass: } U &= mg(L(t) - L(t)Cos(\theta(t))) \\
\text{Linear Spring: } U &= \frac{k(L(t) - Leq)^2}{2} \\
\text{Torsional Spring: } U &= \frac{k_2(\theta(t))^2}{2}
\end{aligned}$$

Kinetic Energy

$$\begin{aligned}
\text{Mass as a swinger: } T &= \frac{mL(t)^2\dot{\theta(t)}^2}{2} \\
\text{Mass as a Slider: } T &= \frac{m(\dot{L}(t))^2}{2}
\end{aligned}$$

Defining the Lagrangian $La = T - U$, taking the partial derivatives we have:

$$\begin{aligned}
\frac{\partial La}{\partial \dot{\theta}} &= mL(t)^2\dot{\theta}(t) \\
\frac{\partial La}{\partial \theta} &= -gmL(t)\sin(\theta(t)) - k_2\theta(t) \\
\frac{\partial La}{\partial \dot{L}} &= m\dot{L}(t) \\
\frac{\partial La}{\partial L} &= kLeq - gm + gm\cos(\theta(t)) + L(t)\left(m\dot{\theta}(t)^2 - k\right)
\end{aligned}$$

To obtain the ODE, we utilize the relation: $\frac{d}{dt}\frac{\partial La}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$, and $\frac{d}{dt}\frac{\partial La}{\partial \dot{L}} - \frac{\partial L}{\partial L} = 0$

$$\begin{aligned}
\ddot{\theta} &= -\frac{g\sin\theta}{L} - \frac{2\dot{L}\dot{\theta}}{L} - \frac{k_2\theta}{mL^2} \\
\ddot{L} &= \frac{k(Leq - L) - mg}{m} + \frac{mg\cos\theta + mL\dot{\theta}^2}{m}
\end{aligned}$$