For your reference only. You are responsible for checking the correctness of all formulae.

Chapter One. Free vibration of 1-DOF systems

Natural frequency, $\omega_n = \sqrt{k/m}$, period $T = 2\pi/\omega_n$

Free response of 1-DOF undamped systems $x = A\sin(\omega_n t + \phi),\ A = \dfrac{\sqrt{\omega_n^2 x_0^2 + v_0^2}}{\omega_n},\ \phi = \tan^{-1}\dfrac{\omega_n x_0}{v_0}$

Damping ratio $\varsigma = \dfrac{c}{c_{cr}} = \dfrac{c}{2\sqrt{km}}$, damped natural frequency $\omega_d = \omega_n\sqrt{1-\varsigma^2}$, Period $T = \dfrac{2\pi}{\omega_d}$,

Free response of underdamped systems

$$x = Ae^{-\varsigma\omega_n t}\sin(\omega_d t + \phi),\ A = \frac{\sqrt{(v_0 + \varsigma\omega_n x_0)^2 + (x_0\omega_d)^2}}{\omega_d},\ \phi = \tan^{-1}\frac{x_0\omega_d}{v_0 + \varsigma\omega_n x_0}$$

Stiffness, in general, $k = F/\triangle l$ or $M/\triangle\theta$, a cantilever beam $k = 3EI/l^3$, axial stiffness of a rod $k = EA/l$,

Torsional stiffness of a rod $k = GJ/l$,

Stiffness of two springs in parallel $k = k_1 + k_2$, stiffness of two springs in series $k = k_1 k_2/(k_1 + k_2)$

Logarithmic decrement $\delta = \ln\dfrac{x(t)}{x(t+T)}$, damping ratio $\varsigma = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$

Chapter Two. Response to Harmonic Excitation $F(t) = F_0\cos\omega t$ (note $f_0 = F_0/m$)

Response of undamped system $x = \dfrac{v_0}{\omega_n}\sin\omega_n t + (x_0 - \dfrac{f_0}{\omega_n^2 - \omega^2})\cos\omega_n t + \dfrac{f_0}{\omega_n^2 - \omega^2}\cos\omega t,\ \omega \neq \omega_n$

Frequency of beat $\omega_{beat} = |\omega_n - \omega|$, frequency of oscillation $\omega_{oscil} = \dfrac{\omega_n + \omega}{2}$

Response of underdamped system to a harmonic force $F = F_0\cos\omega t$

$$x = Ae^{-\varsigma\omega_n t}\sin(\omega_d t + \phi) + X\cos(\omega t - \theta),\ X = \frac{F_0/m}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\varsigma\omega_n\omega)^2}},\ \theta = \tan^{-1}\frac{2\varsigma\omega_n\omega}{\omega_n^2 - \omega^2}$$

$$\phi = \tan^{-1}\frac{\omega_d(x_0 - X\cos\theta)}{v_0 + (x_0 - X\cos\theta)\varsigma\omega_n - \omega X\sin\theta},\ A = \frac{x_0 - X\cos\theta}{\sin\phi}$$

Base excitation $y = Y\sin\omega_b t$,

Displacement transmissibility $\dfrac{X}{Y} = \left[\dfrac{1 + (2\varsigma r)^2}{(1-r^2)^2 + (2\varsigma r)^2}\right]^{1/2},\ r = \dfrac{\omega_b}{\omega_n}$

Force transmissibility $\dfrac{F_T}{kY} = r^2\left[\dfrac{1 + (2\varsigma r)^2}{(1-r^2)^2 + (2\varsigma r)^2}\right]^{1/2}$

Rotating Unbalance. Magnitude of steady-state response $X = \dfrac{m_0 e}{m}\dfrac{r^2}{\sqrt{(1-r^2)^2 + (2\varsigma r)^2}},\ r = \dfrac{\omega}{\omega_n}$

Chapter 3. General Forced Response

Response to an impulse at $t=0$, $F(t)=\hat{F}\delta(t)$, $x(t)=\dfrac{\hat{F}}{m\omega_d}e^{-\varsigma\omega_n t}\sin\omega_d t$

Response to an impulse at $t=\tau$, $F(t)=\hat{F}\delta(t-\tau), t\geq\tau$, $x(t)=\dfrac{\hat{F}}{m\omega_d}e^{-\varsigma\omega_n(t-\tau)}\sin\omega_d(t-\tau), t\geq\tau$

Response to a step input at $t=0$, $F(t)=F_0$, $x(t)=\dfrac{F_0}{k}-\dfrac{F_0}{k\sqrt{1-\varsigma^2}}e^{-\varsigma\omega_n t}\cos(\omega_d t-\theta)$, $\theta=\tan^{-1}\dfrac{\varsigma}{\sqrt{1-\varsigma^2}}$

Response to a step input at $t=\tau$, $F(t)=F_0$, $x(t)=\dfrac{F_0}{k}-\dfrac{F_0}{k\sqrt{1-\varsigma^2}}e^{-\varsigma\omega_n(t-\tau)}\cos[\omega_d(t-\tau)-\theta], t\geq\tau$

Chapter 4. Multiple-Degree-Freedom Systems

Matrix equation of undamped systems free of excitation $M\ddot{\mathbf{x}}+K\mathbf{x}=\mathbf{0}$

Natural frequencies can be found by solving the equation $\det\left[-\omega^2 M+K\right]=0$

Mode shapes can be found by solving the equation $\left[-\omega_i^2 M+K\right]\mathbf{u}_i=\mathbf{0}$

Free response using the modal analysis method

1. Calculate $M^{-1/2}$
2. Calculate $\tilde{K}=M^{-1/2}KM^{-1/2}$
3. Solve for the eigenvalues $\det\left[\tilde{K}-\omega^2 I\right]=0$ and eigenvectors $\left[\tilde{K}-\omega_i^2 I\right]\mathbf{v}_i=\mathbf{0}$
4. Normalize $\overline{\mathbf{v}}_i=\mathbf{v}_i/\sqrt{\mathbf{v}_i^T\mathbf{v}_i}$ and form the matrix $P=\left[\overline{\mathbf{v}}_1 \quad \overline{\mathbf{v}}_2\right]$
5. Calculate the matrix $S=M^{-1/2}P$ and $S^{-1}=P^T M^{1/2}$
6. Calculate the initial conditions in the modal coordinates $\mathbf{r}_0=S^{-1}\mathbf{x}_0$, $\dot{\mathbf{r}}_0=S^{-1}\dot{\mathbf{x}}_0$
7. Find the response in the modal coordinates $\ddot{r}_i+\omega_i^2 r_i=0, r_{i0}, \dot{r}_{i0}$, $r_i=\sqrt{r_{i0}^2+\dfrac{\dot{r}_{i0}^2}{\omega_i^2}}\sin(\omega_i t+\tan^{-1}\dfrac{\omega_i r_{i0}}{\dot{r}_{i0}})$
8. Find the response in the physical coordinates $\mathbf{x}(t)=S\mathbf{r}(t)$

Proportional damping if $C=\alpha M+\beta K$, damping is proportional. In such case, $S^T CS=diag\left[2\varsigma_i\omega_i\right]$. In the case of a 2-DOF system, $2\varsigma_1\omega_1=\alpha+\beta\omega_1^2$, $2\varsigma_2\omega_2=\alpha+\beta\omega_2^2$

Free response of a damped system. Solve the response in the modal coordinate $\ddot{r}_i+2\varsigma_i\omega_i\dot{r}_i+\omega_i^2 r_i=0, r_{i0}, \dot{r}_{i0}$. Then transform it back to the physical coordinate $\mathbf{x}(t)=S\mathbf{r}(t)$.

Forced response of an undamped system. Find the force in the modal coordinate $\mathbf{f}(t)=S^T\mathbf{F}(t)$. Solve for r_{ih} and r_{ip} from the equation: $\ddot{r}_i+\omega_i^2 r_i=f_i, r_{i0}, \dot{r}_{i0}$. The formula for r_i can be found in Chapter two. Then transform it back to the physical coordinate $\mathbf{x}_p=S\mathbf{r}_p$ and $\mathbf{x}_h=S\mathbf{r}_h$. $\mathbf{x}=\mathbf{x}_p+\mathbf{x}_h$.

Forced response of a damped system. Find the force in the modal coordinate $\mathbf{f}(t)=S^T\mathbf{F}(t)$. Solve for r_{ip} and r_{ih} from the equation: $\ddot{r}_i+2\varsigma_i\omega_i\dot{r}_i+\omega_i^2 r_i=f_i, r_{i0}, \dot{r}_{i0}$. Then transform it back to the physical coordinate $\mathbf{x}_p=S\mathbf{r}_p$ and $\mathbf{x}_h=S\mathbf{r}_h$. $\mathbf{x}=\mathbf{x}_p+\mathbf{x}_h$.

Chapter Five. Design for Vibration Suppression.
Understand the use of Nomograph for specifying acceptable limits of sinusoidal vibration (also refer to Section 1.2)
Vibration isolation

Displacement transmissibility $\frac{X}{Y} = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2} \right]^{1/2}$ for isolating a device from a vibrating source.

Force transmissibility $\frac{F_T}{F_0} = \left[\frac{1+(2\varsigma r)^2}{(1-r^2)^2+(2\varsigma r)^2} \right]^{1/2}$ for isolating a vibrating source from its surroundings.

Vibration absorber. The principle $\omega_a = \omega$ such that $X = 0$.

$$\frac{Xk}{F_0} = \left| \frac{1-\omega^2/\omega_a^2}{[1+\mu(\omega_a/\omega_p)^2-(\omega/\omega_p)^2][1-(\omega/\omega_a)^2]-\mu(\omega_a/\omega_p)^2} \right|$$

where $\omega_a = \sqrt{k_a/m_a}$, $\omega_p = \sqrt{k/m}$, $\mu = m_a/m$, $\beta = \omega_a/\omega_p$

Natural frequencies of the entire system can be found by solving

$$[1+\mu(\omega_a/\omega_p)^2-(\omega/\omega_p)^2][1-(\omega/\omega)^2]-\mu(\omega_a/\omega_p)^2 = 0$$

Operation range can be found by solving

$$\frac{Xk}{F_0} = \pm 1$$

Basics of linear algebra

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix},\ A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix},\ \det(A) = ad - bc,\ A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$