Department of Aerospace Engineering, IIT Madras
AS 303 Vibrations (Aug- Dec 2006)

Problem Set 3
(Due by 8.11.2006)

1. (a) Using the extended Hamilton's principle, obtain the governing equation together with the boundary conditions for the free longitudinal vibrations of a prismatic rod with a lumped mass *m* at one end and fixed at the other end. Assume the mass density and stiffness to be constant along the length of the bar.

   (b) Show that the natural frequencies of longitudinal vibrations are given by the equation

$$\omega l\sqrt{\frac{\rho}{E}}\,\tan\left(\omega l\sqrt{\frac{\rho}{E}}\right) = \frac{\rho A l}{m}$$

2. Derive the governing equation and the boundary conditions for the free transverse vibrations of a cantilever beam with a lumped mass *m* at one end, using the extended Hamilton's principle. Assume that the rigidity *EI* and the mass per unit length *m* are varying along the length of the beam.

3. Effect of Axial Loading: The presence of axial loading can have a significant effect on the transverse vibrations of a beam. Tension in a beam increases its stiffness to transverse motion and therefore increases its natural frequencies. Show that the governing equation for transverse vibrations gets modified to

$$EI\,\frac{\partial^4 y}{\partial x^4} - T\,\frac{\partial^2 y}{\partial x^2} + \rho A\,\frac{\partial^2 y}{\partial t^2} = 0$$