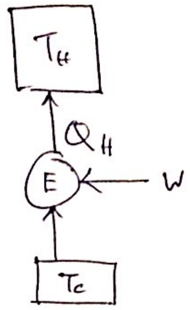
Tutorial 5 Solutions

1.

$T_{amb} = 35°C = 308\,K$ $\qquad$ $T_{room} = 20°C$

$V_{house} = 540\,m^3 \times 100$



$$Q_c = V_{house} \frac{P}{RT} \times C_p \Delta T$$

$$= \frac{540 \times \cancel{100} \times 101325 \times 15 \times 1010}{288.6 \times \underbrace{300}_{\text{average}}} \; J$$

$$Q_H = Q_c + W$$

$= 9.57\,MJ$

COP of ACs $= 3$

$$W = \frac{Q_c}{3} = 3.19\,MJ$$

$Q_H = 12.77\,MJ$

given to 35°C air in volume $(9290 \times 15 - 70000)\,m^3$

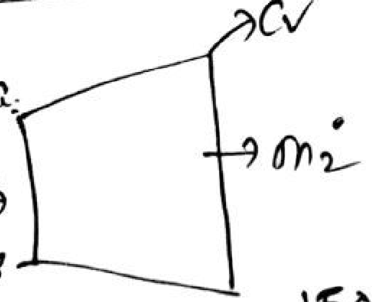
$$\rho_{out} = \frac{P}{RT} = \frac{101325}{288.6 \times 308} = 1.139\ kg/m^3$$

$$M_{out} = 1.132 \times 85354 = 97218\ kg$$

$$\Delta T = \frac{Q_c}{M_{out}\,C_p} = \frac{12.77 \times 10^6 \times 100}{97218 \times 1010}$$

$= 13\,K$

2) 100 kPa
27°C
6 kg/s
60 m/s
h = 900 kJ/kg



150 m/s
400 kJ/kg

$\dot{Q} = -28$ kJ/kg

Power output = ??

$$\dot{Q} - \dot{W} = \frac{dE_{cv}}{dt} + \dot{m}_2\left(h_2 + \frac{u_2^2}{2} + gz_2\right) - \dot{m}_1\left(h_1 + \frac{u_1^2}{2} + gz_2\right)$$

Neglecting potential energy changes.

S State

$$-28\times 10^3 \times \dot{m} - \dot{W} = \dot{m}\left(h_2 - h_1 + \frac{u_2^2 - u_1^2}{2}\right)$$

$$\dot{W} = \left[-28\times 10^3 - 400\times 10^3 + 900\times 10^3 - \frac{150^2}{2} + \frac{60^2}{2}\right] \times 6$$

$\dot{W}$ = ~~3111300~~ 2775300 Watts

Power out = ~~3111.3~~ 2775 kW

(3)

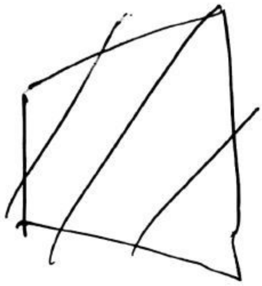
0.1 MPa.

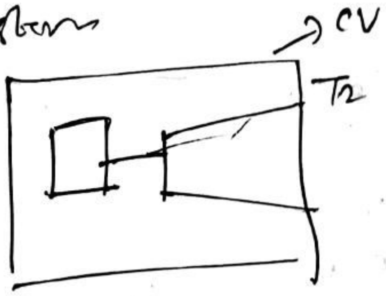
$W = Q - E$

$V = 0.03 m^3$
$P = 3.5 MPa$
$T = 400\,°C$

Find work done by the fluid on Turbine.

Lets consider the whole thing as system





$$\dot{Q}^0 - \dot{W} = \frac{dE_{cv}}{dt} + \dot{m}_{out}\left(h_{out} + \frac{u^2}{2} + gz\right)$$

Neglecting heat transfer

$$-\dot{W} = \frac{dE_{cv}}{dt} + \dot{m}_{out}\,(C_p)\,T_2$$

Neglecting the volume occupied by Turbine.

$T_2 \Rightarrow$ atmospheric temperature (constant)

Integrate

~~$-W = E_2 - E_1$~~

$-W = E_f - E_i + M_{out}\,(C_p)\,T_2$

$-W = m_f C_v T_f - m_i C_v T_i + (m_f - m_i)\, C_p T_2$

$T_i = 400°C$

Adiabatic expansion $\Rightarrow$

$$\frac{(400+273)^{\gamma}}{(3.5\times10^6)^{\gamma-1}} = \frac{T_2^{\gamma}}{(0.1\times10^6)^{\gamma-1}}$$

$\Rightarrow$ $T_2 = 243.6981\,K$

Airflow stops when pressure difference becomes zero.

$(P_f)_{cv} = P_2 = 0.1$ ~~atm~~ MPa.

⇒ Final temperature

$$\frac{((T_f)_{cv})^{\gamma}}{(0.1)^{\gamma-1}} = \frac{(T_2)^{\gamma}}{(0.1)^{\gamma-1}}$$

$$\Rightarrow (T_f)_{cv} = T_2 = 243.7$$

$$m_f = (\rho) V$$
$$= \frac{P}{RT} V$$

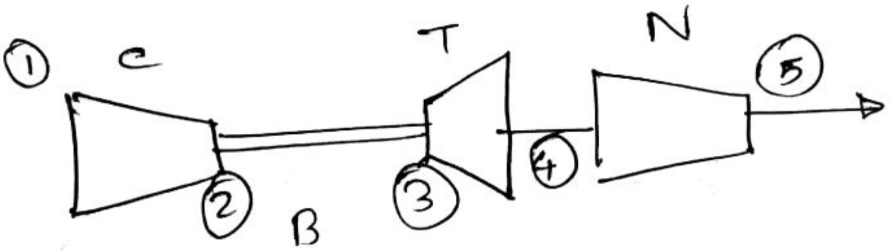
$$m_f = \frac{0.1 \times 10^6}{288 \times 243.7} \times 0.3 = 0.427 \text{ kg}$$

$$m_i = \frac{3.5 \times 10^6}{288 \times 673} \times 0.3 = 5.417 \text{ kg}$$

$$-W = (0.427) \times 717.85 \times 243.7 - 5.417 \times 717.85 \times 673$$
$$- (0.427 - 5.417)(1005)(243.7)$$

$$W = 1317.384 \text{ kJ}$$

(4)



$\dot{m}_a = 100$ kg/s

$T_1 = T_a = 300$ K

$P_1 = 1$ atm

$P_2 = 18$ atm

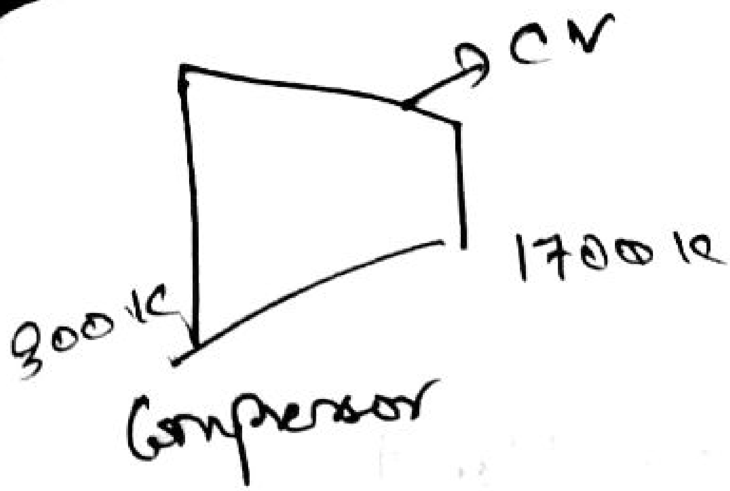
$T_3 = 1700$ K

$$\dot{Q} - \dot{W} = \frac{dE_{CV}}{dt} + \dot{m}_{out}\left(h_{out} + \frac{u^2}{2} + gz_{out}\right) - \dot{m}_{in}\left(h_{in} + \frac{u_{in}^2}{2} + gz_{in}\right)$$

~~$T_5 = ??$~~

$T_4 = ??$

To get T_4 we need the enthalpy of gas at 4. Which requires Work extracted by Turbine that depends on Compressor



Neglecting the inlet velocity

isentropic compression $\Rightarrow$ $\dot{Q} = 0$

steady state $\Rightarrow$ $\frac{dE}{dt} = 0$

T_2 is not known!!

Assuming isentropic compression

$$pv^{\gamma} = \text{Constant}$$

$$p = \rho RT$$

$$p\left(\frac{RT}{p}\right)^{\gamma} = \text{Constant}$$

$$\frac{T_1^{\gamma}}{p_1^{\gamma-1}} = \frac{T_2^{\gamma}}{p_2^{\gamma-1}}$$

$$(300)^{\gamma} \times \left(\frac{18}{1}\right)^{\gamma-1} = T_2^{\gamma}$$

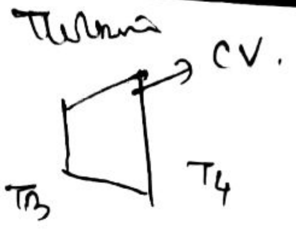
$$\gamma = 1.4$$

$$T_2 = 685 \text{ K}$$

$$-\dot{W} = \dot{m}(h_2 - h_1)$$ ~~100~~

$$\dot{W} = -100 \times 1005 \times (685 - 300) = -38.693 \text{ MW}$$

Turbine



$T_3 = 1700\,K$.

Work done by Compressor on Fluid

= work done by Fluid on Turbine

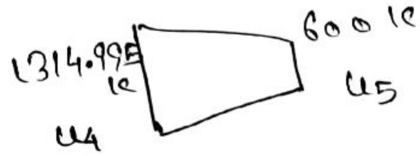
$-\dot{W}_{\text{Fluid on turbine}} = \dot{m} C_p (T_4 - T_3)$

$= 100 \times 1005\,(T_4 - 1700) = -38.693\times10^6$

$T_4 = 1700 - \dfrac{38.693\times10^6}{100\times1005}$

$T_4 = 1314.995\,K$

Nozzle



Assume $u_4 << u_5$

$\dot{Q} = 0$

$\dot{W} = 0$

Steady state.

$h_4 + \dfrac{u_4^2}{2} = h_5 + \dfrac{u_5^2}{2}$

$C_p (T_4 - T_5) = \dfrac{u_5^2}{2}$

$u_5 = \sqrt{2(1005)(1314.995 - 600)}$

$u_5 = 1198\ m/s$

5)

$$W_1 = W_2$$

$$Q_h - Q = Q - Q_c$$

$$\frac{Q_h}{Q} - 1 = 1 - \frac{Q_c}{Q}$$

$$\frac{T_h}{T} - 1 = 1 - \frac{T_c}{T} \Rightarrow \boxed{T = \frac{T_{H} + T_c}{2}}$$

(b) when $\eta_1 = \eta_2$

$$1 - \frac{Q}{Q_h} = 1 - \frac{Q_c}{Q}$$

$$1 - \frac{T}{T_h} = 1 - \frac{T_c}{T}$$

$$\Rightarrow \quad T = \sqrt{T_c T_h}$$