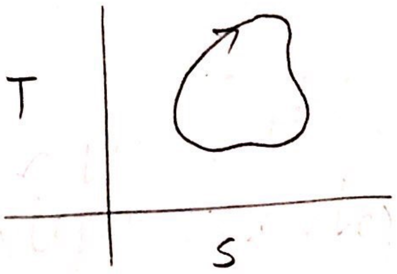
Tutorial 6

1)

(a) for a cyclic process, I law states that $\oint \delta Q = \oint \delta W$



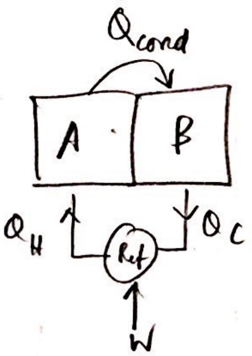
$$\oint_{cycle} \delta Q = \oint T ds = \text{area in the cycle}$$

$$\oint \delta W = \oint P dV = \text{area in the PV diagram}$$

∴ the two areas are equal!

(b) it is true for any cycle so it is true for refrigiration cycle as well

2)



$$\dot{Q}_{cond} = -KA \frac{dT}{dx} = \frac{KA(T_A - T_B)}{S_u}$$

for steady state,

$$\dot{Q}_{cond} = Q_c$$

for minimum work

reversible refrigirator $\Rightarrow \dot{Q}_H = \frac{\dot{Q}_c}{T_B} T_A$

$$\dot{W} = \dot{Q}_H - \dot{Q}_C$$

$$\therefore \dot{W} = \dot{Q}_H - \dot{Q}_c$$

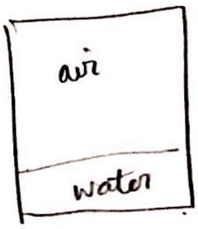
$$= \dot{Q}_c\left(\frac{T_A}{T_c} - 1\right) = \frac{\dot{Q}_{cond}}{(COP)_{ref}}$$

$$\dot{W} = \frac{KA(T_A - T_B)}{(COP)_{ref}}$$

$$\Delta S = \Delta S_A + \Delta S_B + \cancelto{0}{\Delta S_{ref}} = \cancelto{0}{\frac{\dot{Q}_H}{T_A} - \frac{\dot{Q}_c}{T_B}} - \frac{\dot{Q}_{cond}}{T_A} + \frac{\dot{Q}_{cond}}{T_B}$$

$$= \dot{Q}_{cond}\left[\frac{1}{T_B} - \frac{1}{T_A}\right] = \frac{\dot{W}}{T_A}$$

3.



air $V = 0.2\,m^3$ 25°C 1 bar

water $m = \frac{1}{2}$ Kg

2 gms evaporated

$$n_{air} = \frac{PV}{RT} = \frac{10^5 \times 0.2}{8.314 \times 288} = 8.07 \text{ moles}$$

$$m_{air} = 28.8 \times 10^{-3} \times 8.07 = 0.232 \text{ Kg}$$

$$n_w = \frac{2\times10^{-3}}{18} = 0.111 \text{ mol}$$

$$X_w = \frac{n_w}{n_a + n_w} = \frac{0.111}{8.07 + 0.11} = 0.013$$

$$Y_w = \frac{m_w}{m_a + m_w} = \frac{2\times10^{-3}}{2\times10^{-3} + 0.232} = 0.00854$$

4) mass %

$CH_4 = 58\%$ $\quad Cp_{CH_4} = 35.06$ kJ/kmol K

$H_2 = 18\%$ $\quad Cp_{H_2} = 29.01$ "

$N_2 = 10\%$ $\quad Cp_{N_2} = 28.87$ "

$CO = 14\%$ $\quad Cp_{CO} = 28.88$ "

(a) $$\frac{1}{M_{w_m}} = \sum_i \frac{Y_i}{M_{w_i}} = \frac{0.58}{16} + \frac{0.18}{2} + \frac{0.1}{28} + \frac{0.14}{28} = 0.135$$

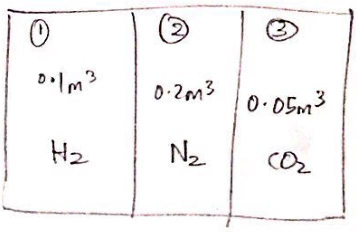
(b) $$R_m = \frac{8.314}{7.42} = 1120 \text{ J/kgK}$$

(c) $$C_{p_m} = \sum_i Y_i C_{p_i} = \sum_i Y_i \hat{C}_{p_i}$$

$$= \frac{0.58}{16} \times 35.06 + \frac{0.18}{2} \times 29.01 + \frac{0.1}{28} \times 28.87 + \frac{0.14}{28} \times 28.88 = 4.129 \frac{\text{kJ}}{\text{kg K}}$$

$$C_{v_m} = C_{p_m} - R_m = 4.129 - 1.12 = 3.009 \text{ kJ/kgK}$$

5.



$\forall_{total} = 0.35m^3$

$P = 2$bar $\quad T = 13°C = 286K$

<u>mixing</u>: $$\eta_{H_2} = \frac{P\forall_{H_2}}{RT} = \frac{2 \times 10^5 \times 0.1}{8.314 \times 286} = 8.41 \text{ moles}$$

$$\eta_{N_2} = \frac{2 \times 10^5 \times 0.2}{8.314 \times 286} = 16.82 \text{ moles}$$

$$\eta_{CO_2} = \frac{2 \times 10^5 \times 0.05}{8.314 \times 286} = 4.205 \text{ moles}$$

$X_{H_2} = \frac{8.41}{29.44} = 0.286$

$X_{N_2} = 0.572$

$Y_{CO_2} = 0.143$

a) $M_m = \sum X_i M_i$

$= 0.286 \times 2 + 0.572 \times 28$

$+ 0.143 \times 43$

$= 22.88 \, kg/kmol$

b) $R_m = \frac{8.314}{22.88} = 363.4 \, J/KgK$

c) $P_i = X_i P \longrightarrow P_{H_2} = 0.572 \, bar \, ; P_{N_2} = 1.144 \, bar; \, P_{CO_2} = 0.286 \, bar$

d) $$Cp_{mix} = \sum_i X_i \hat{Cp}_i = \sum_i X_i \, Cp_i M_i$$

$$= 0.286 \times 2 \times 14.235 + 0.572 \times 28 \times 1.039$$

$$+ 0.143 \times 0.828 \times 44$$

$$= 29.99 \, kJ/Kmol \, K$$

$$Cv_{mix} = Cp_{mix} - R_{mix} = 29.99 - 8.314 = 21.676 \, kJ/kmol \, K$$

$\gamma = 1.384$

e) each gas occupies the full volume during mixing through free expansion

$$\Delta S = \cancel{C_V \ln\left(\frac{T_2}{T_1}\right)}^{\to 0} + R \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S_{H_2} = \frac{8.314 \times \cancel{10^{+3}}}{2} \ln\left(\frac{0.35}{2}\right) = 5.21 \, kJ/KgK$$

$$\Delta S_{N_2} = \frac{8.314}{28} \ln\left(\frac{0.35}{0.2}\right) = 0.166 \, kJ/KgK$$

$$\Delta S_{CO_2} = \frac{8.314}{44} \ln\left(\frac{0.35}{0.05}\right) = 0.367 \, \frac{KJ}{KgK}$$

$$\Delta S_{mix} = \left[0.286 \times 2 \times 5.21 + 0.572 \times 28 \times 0.166 + 0.143 \times 44 \times 0.367\right] \frac{1}{22.88}$$

$$= 0.3474 \ \frac{kJ}{kg\,K}$$

Compression:

$PV^{1.2} = Const.$

Work for $PV^{n} = const$ process : $W = \int_1^2 PdV = \frac{P_1V_1 - P_2V_2}{n-1}$

$$V_2 = \left(\frac{P_1}{P_2}\right)^{1/1.2} V_1$$

$$= \left(\frac{2}{6}\right)^{1/1.2} \times 0.35$$

$$= 0.14 m^3$$

$$W = \left[\frac{2 \times 0.35 - 6 \times 0.14}{0.2}\right] \times 10^5 = -70 \text{ kJ}$$

$$T_2 = \frac{P_2V_2}{nR} = \frac{6 \times 10^5 \times 0.14}{29.44 \times 8.314} = 343.4 \text{ K}$$

$$\Delta Q = \Delta E + \delta W = 21.676 \frac{J}{mol K} \times 29.44 (343.2 - 286) - 70$$

$$= -33.5 \text{ kJ}$$

g) $$\Delta S = N\left(\hat{C}p \ln\left(\frac{T_2}{T_1}\right) - \hat{R} \ln\left(\frac{P_2}{P_1}\right)\right)$$

$$= 29.44 \left[29.99 \ln\left(\frac{343.2}{286}\right) - 8.314 \ln\left(\frac{6}{2}\right)\right] = -107.9 J/K$$