Q2 Contd

$$\therefore \quad r_p^{\frac{2(1-\gamma)}{\gamma}} = \frac{T_{min}}{T_{max}}$$

$$\text{or} \quad \boxed{r_p)_{opt} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{(\gamma-1)2}} = \sqrt{r_{p_{max}}}}$$

This is the reason why most engines now operate at ~~[illegible]~~ $r_p = 30$

$$\left(\sqrt{\frac{1550}{220}}\right)^{3.5} = 30.4$$

$$\frac{\dot{W}_{max}}{\dot{m}} = c_p\left[T_{max}\left(1-\left(\frac{1}{r_{p_{opt}}}\right)^{\frac{\gamma-1}{\gamma}}\right) - T_{min}\left(\left(r_{p_{opt}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)\right]$$

$$= c_p\left[T_{max}\left[1-\left(\frac{T_{min}}{T_{max}}\right)^{1/2}\right] - T_{min}\left[\left(\frac{T_{max}}{T_{min}}\right)^{1/2} - 1\right]\right]$$

$$= c_p\left[\sqrt{T_{max}}\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right) - \sqrt{T_{min}}\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)\right]$$

$$\boxed{\frac{\dot{W}_{max}}{\dot{m}} = c_p\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2}$$

(c) $$\eta_{opt} = \frac{\left(\frac{\dot{W}_{max}}{\dot{m}}\right)}{\left(\frac{\dot{Q}_{opt}}{\dot{m}}\right)} = \frac{\cancel{c_p}\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2}{\cancel{c_p}\left(T_{max} - T_2\right)} = \frac{\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2}{T_{max} - \frac{T_{max}}{\left(r_{p_{opt}}\right)^{\frac{\gamma-1}{\gamma}}}}$$

$$= \frac{\left(\sqrt{T_{max}} - \sqrt{T_{min}}\right)^2}{T_{max}\left(1-\left(\frac{T_{min}}{T_{max}}\right)^{1/2}\right)} = \frac{\sqrt{T_{max}} - \sqrt{T_{min}}}{\sqrt{T_{max}}} = 1 - \sqrt{\frac{T_{min}}{T_{max}}}$$

$$\boxed{\text{or} \quad 1 - \frac{1}{\left(r_{p_{opt}}\right)^{\frac{\gamma-1}{\gamma}}}}$$

(d) $$r_{p_{opt}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{3.5}{2}} = \left(\frac{1600}{220}\right)^{1.75} = \underline{32.2}$$

$$\eta_{opt} = 1 - \sqrt{\frac{220}{1600}} = \underline{0.63}$$

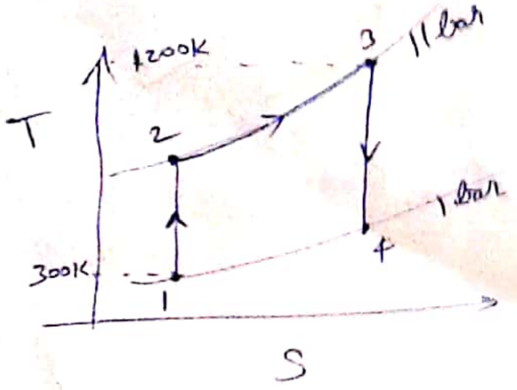
$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 0.86$$

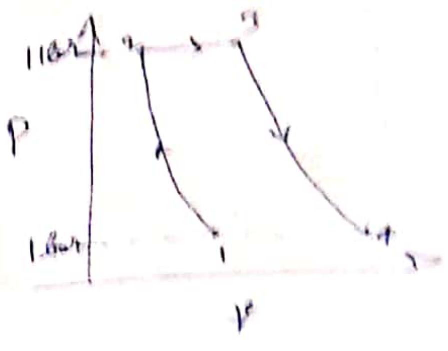
Ans

Q1:

$P_1 = 1\ \text{bar}, \quad T_1 = 300\text{K}.$

$\frac{P_2}{P_1} = 11, \quad T_3 = 1200\text{K}.$





① already given: 1 bar, 300 K $\qquad v_1 = \frac{288.6 \times 300}{10^5} = 0.866\ m^3/kg$

②: we know $P_2 = 11$ bar. $T_2 = ?,\ v_2 = ?$

$1 \to 2$ is isentropic compression $\therefore \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1}{3.5}}$

$\therefore T_2 = 300\,(11)^{\frac{1}{3.5}} = 300 \times 1.98 = 594\ K$

$v_2 = \frac{RT_2}{P_2} = \frac{288.6 \times 594}{11 \times 10^5} = 0.156\ m^3/kg$

③: given $P_3 = 11$ bar & $T_3 = 1200$K.

$v_3 = \frac{RT_3}{P_3} = 0.315\ m^3/kg$

$\dot{Q}_{in} - \dot{W}_{in} = \dot{m}(h_3 - h_1)$

$\frac{\dot{Q}_{in}}{\dot{m}}$ = heat in per mass $= h_3 - h_2 = 3.5 \times 288.6 \times (1200 - 594)$

$= 612.1\ kJ/kg$

④ $P_4 = 1$ bar, $v_4 = ?\ T_4 = ?$

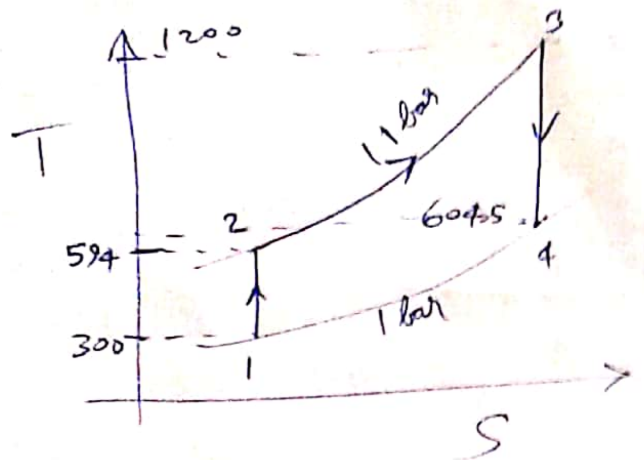
$3 \to 4$ isentropic $\therefore \frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \therefore T_4 = 1200 \times \left(\frac{1}{11}\right)^{\frac{1}{3.5}} = 604.8\ K$

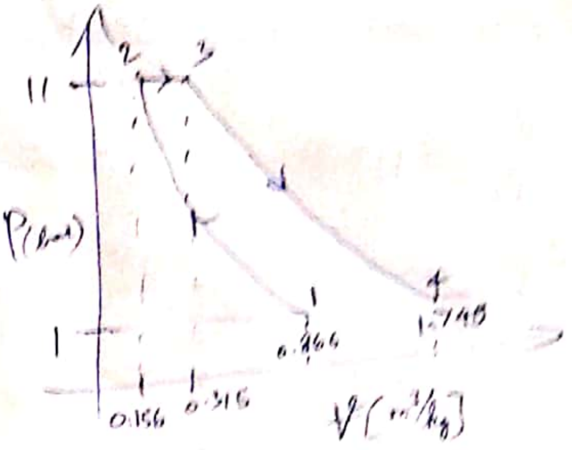
$v_4 = \frac{RT_4}{P_4} = 1.745\ m^3/kg$

Percentage increase due to regeneration

$$= \frac{0.2837 - 0.199}{0.199} = \mathbf{0.4256, \text{ or } 42.56\%}$$

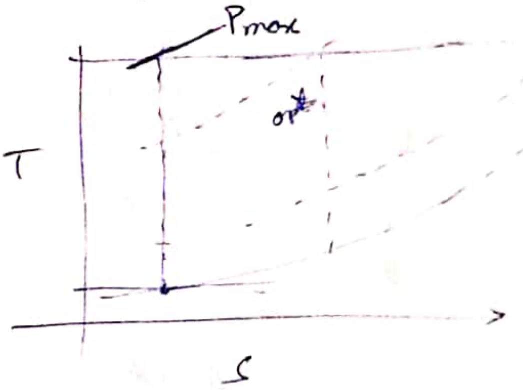
So the actual cycle diagram looks like





Q2 : Solns to Tut 7

(a) given T_{max} & T_{min} find max pressure ratio to operate Brayton cycle



$$r_{p_{max}} = \frac{P_{max}}{P_{min}} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{\gamma-1}}$$

(b) ~~η vs r_p~~ $W_{net} = Q_{net} = C_p(T_3 - T_2) - C_p(T_4 - T_1)$

$$= C_p\left[T_{max} - T_2 - T_4 + T_{min}\right]$$

$$T_2 = T_{min}(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T_4 = T_{max}\left(1/r_p\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore W_{net} = C_p\left[T_{max}\left(1 - \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}\right) - T_{min}\left((r_p)^{\frac{\gamma-1}{\gamma}} - 1\right)\right]$$

for optimum,

$$\frac{dW_{net}}{dr_p} = C_p\left[T_{max}\left(+\left(\frac{\gamma-1}{\gamma}\right)(r_p)^{-\left(\frac{\gamma-1}{\gamma}\right)-1}\right) - T_{min}\left(\left(\frac{\gamma-1}{\gamma}\right)r_p^{\left(\frac{\gamma-1}{\gamma}\right)-1}\right)\right] = 0$$

$$\therefore r_p^{\left[\frac{1-2\gamma}{\gamma} - \frac{\gamma-1-\gamma}{\gamma}\right]} = \frac{T_{min}}{T_{max}}$$

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Q3 soln to tut-7

ans in pg 242 of PK Nag's solns Q.13.29 (I will send photo)

I modified only the last part

our answer will be $\frac{4272 kW}{40 kg/s} \sim 100 kJ/kg$

Q4 (You guys solve on board)

example 13.9 in book but anyways I will send photo.

Ⓜ Example 13.9 *In a gas turbine plant, working on the Brayton cycle with a regenerator of 75% effectiveness, the air at the inlet to the compressor is at 0.1 MPa, 30°C, the pressure ratio is 6, and the maximum cycle temperature is 900°C. If the turbine and compressor have each an efficiency of 80%, find the percentage increase in the cycle efficiency due to regeneration.* **[LO 13.9]**

Solution *Given:* (Fig. 13.44)

$p_1 = 0.1$ MPa

$T_1 = 303$ K

$T_3 = 1173$ K

$r_p = 6,\ \eta_T = \eta_C = 0.8$

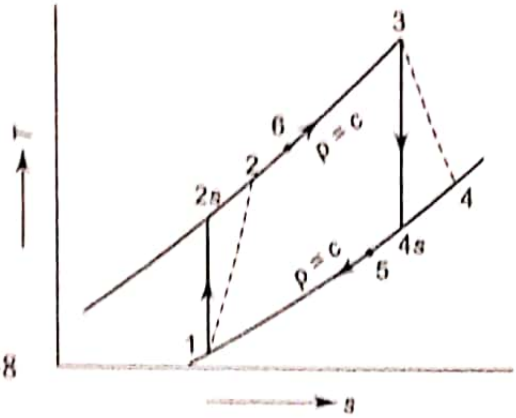

Fig. 13.44

Without a regenerator

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = \frac{T_3}{T_{4s}} = (6)^{0.4/1.4} = 1.668$$

$$T_{2s} = 303 \times 1.668 = 505 \text{ K}$$

$$T_{4s} = \frac{1173}{1.668} = 705 \text{ K}$$

$$T_2 - T_1 = \frac{T_{2s} - T_1}{\eta_C} = \frac{505 - 303}{0.8} = 252 \text{ K}$$

$$T_3 - T_4 = \eta_T (T_3 - T_{4s}) = 0.8\,(1173 - 705) = 375 \text{ K}$$

$$W_T = h_3 - h_4 = c_p (T_3 - T_4) = 1.005 \times 375 = 376.88 \text{ kJ/kg}$$

$$W_C = h_2 - h_1 = c_p (T_2 - T_1) = 1.005 \times 252 = 253.26 \text{ kJ/kg}$$

$$T_2 = 252 + 303 = 555 \text{ K}$$

$$Q_1 = h_3 - h_2 = c_p (T_3 - T_2) = 1.005\,(1173 - 555) = 621.09 \text{ kJ/kg}$$

$$\therefore \quad \eta = \frac{W_T - W_C}{Q_1} = \frac{376.88 - 253.26}{621.09} = 0.199 \text{ or } 19.9\%$$

With regenerator

$$T_4 = T_3 - 375 = 1173 - 375 = 798 \text{ K}$$

$$\text{Regenerator effectiveness} = \frac{T_6 - T_2}{T_4 - T_2} = 0.75$$

$$\therefore \quad T_6 - 555 = 0.75\,(798 - 555)$$

or $$T_6 = 737.3 \text{ K}$$

$$\therefore \quad Q_1 = h_3 - h_6 = c_p (T_3 - T_6)$$

$$= 1.005\,(1173 - 737.3) = 437.88 \text{ kJ/kg}$$

W_{net} remains the same.

$$\therefore \quad \eta = \frac{W_{net}}{Q_1} = \frac{123.62}{437.9} = \mathbf{0.2837 \text{ or } 28.37\%}$$