AS1300 Tut 8 16/APR/2019

Q1 Soln.

$\dot{Q}_{cv} = \dot{m}(\Delta H)$ for const pressure process

$$\therefore \frac{\dot{Q}}{\dot{m}} = \cancel{H_2 - H_1}\ h_2 - h_1$$

$$= -\left[\Delta H_f + \left(-2.857 + \frac{0}{1.455 \times \frac{20}{50}} + 0.582\right)\right]_1 + \left[\Delta H_f + 74.296\right]_2$$

$$= \left\{74.296 + 3.439\right\} \frac{kJ}{mol} \times \frac{1}{28.013\ kg/kmol}$$

$$= 2.775 \times 10^3\ \frac{kJ}{kg}$$

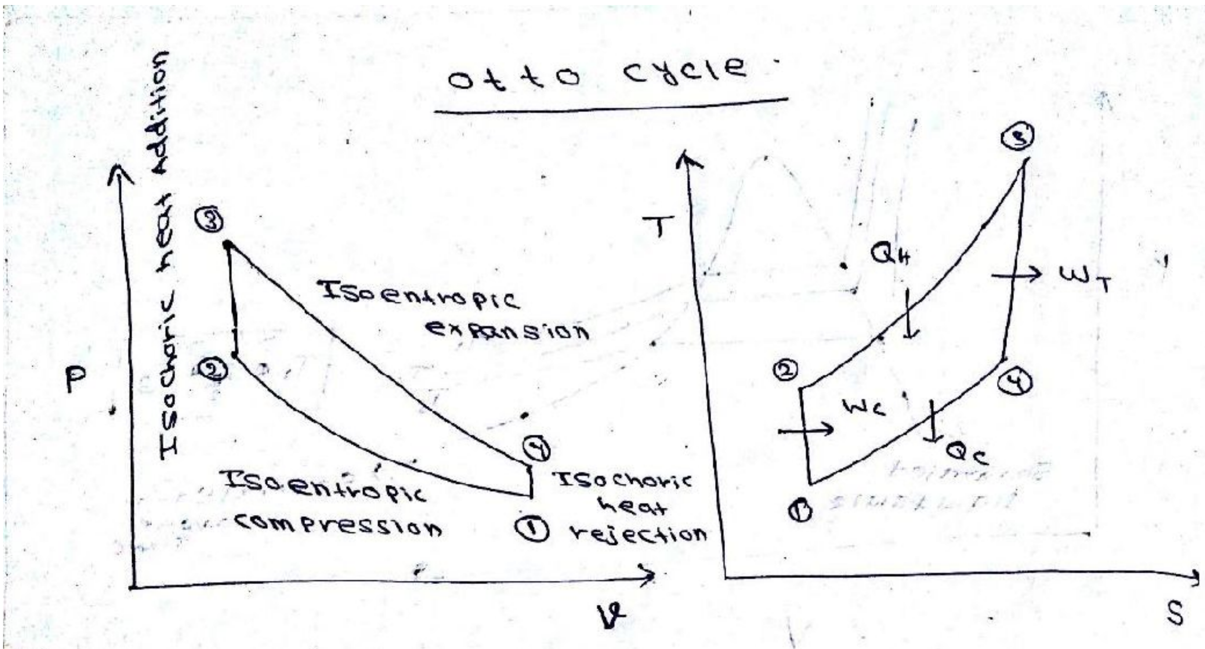
TUTORIAL-8 (16 Apr 2019)
Thermodynamics for Aerospace Engineers (AS1300)
<u>Solutions</u>

1. A process had Nitrogen gas entering a device at 220K and leaving at 2500K. The pressure was maintained at 1 bar throughout. Find the amount of heat added to this gas per kg of gas. Find the entropy change from the tables and from theoretical calculations.

2. Find the heat of formation of CO2 gas at 500K, without using the direct table entry at that temperature. Check your result with the data in the table.

3. An ideal Otto cycle has compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air the constant volume heat addition process. Assuming cold air standard assumption, determine (a) the maximum pressure and temperature that occur during the cycle, (b) the net work done, (c) thermal efficiency and (d) mean effective pressure for the cycle.

<u>Solution</u>



Otto cycle

Q_H = Heat supplied at constant volume

Q_C = Heat rejected at constant volume

W_C = work done in compression Stroke

W_T = work done in power strpke

Fig. 3.1 Reversible Otto cycle

Given: The compression ratio of Otto cycle is 8. The pressure and temperature at the beginning of the compression process 100 kPa and 17°C respectively.

$$\frac{v_1}{v_2} = 8, \quad T_1 = 17°\text{C} = 290\ K, \quad P_1 = 100\ kPa$$

where v is the specific volume (units – m^3/kg)

The process 1-2 shown in the Fig. 3.1 is an isentropic compression process. The isentropic relationship between various state properties can be written as follows:

$$P * v^{\gamma} = constant \qquad [3.1]$$

$$T * v^{\gamma-1} = constant \qquad [3.2]$$

$$\frac{T^{\gamma}}{P^{\gamma-1}} = constant \qquad [3.3]$$

So, using the compression ratio $\left({}^{v_1}/_{v_2}\right)$ the pressure and temperature at the state 2 can be found out.

$$P_2 = P_1 * \frac{v_1}{v_2}^{\gamma} = 100 * 8^{1.4} = 1837.92\ kPa$$

$$T_2 = T_1 * \frac{v_1}{v_2}^{\gamma-1} = 290 * 8^{0.4} = 666.25\ K$$

The process 2-3 shown in Otto cycle shown in the Fig 3.1 is a constant volume or isochoric process. It has been mentioned in the question that 800 kJ/kg amount of heat has been transferred to the system in this process. As the process is a constant volume process, all the heat will be used for changing internal energy of the system without any work done.

Using first law of Thermodynamics:

$$\delta Q = Q_H = \Delta U = C_v * (T_3 - T_2) = 800\ kJ/kg \qquad [3.4]$$

Temperature at state 3 (maximum temperature) can be obtained from equation [3.4] as follows:

$$T_3 = T_2 + \frac{Q_H}{C_v} = 666.25 + \frac{800*1000}{718} = \mathbf{1780.46\ K} \qquad \textbf{Ans}$$

In an isochoric process, temperature is directly proportional to pressure as volume remains constant. So, the maximum pressure (P_3) reached at the end of the isochoric pressure can be calculated as follows:

$$\frac{P_3}{P_2} = \frac{T_3}{T_2} \qquad [3.5]$$

$$P_3 = P_2 * \frac{T_3}{T_2} = 1837.92 * \frac{1780.46}{666.25} = \mathbf{4911.58\ kPa} \qquad \textbf{Ans}$$

Because of the constant volume heat addition and rejection $v_2 = v_3$ & $v_1 = v_4$.

$$\frac{v_1}{v_2} = \frac{v_4}{v_3} = 8 \qquad [3.6]$$

The process 3-4 is the isentropic expansion process. So, the isentropic relationship in equation [3.1] to [3.3] can be used to get temperature and pressure at the 4th state.

$$P_4 = P_3 * \frac{v_3}{v_4}^{\gamma} = 4911.69 * \frac{1}{8}^{1.4} = 267.24\ kPa$$

$$T_4 = T_3 * \frac{v_3}{v_4}^{\gamma-1} = 1780.46 * \frac{1}{8}^{0.4} = 774.99\ K$$

Note: The Otto cycle is a closed cycle. So, the system in this case is considered to be a control mass or closed system. In the isentropic process, $\delta Q = \delta W + \Delta U = 0$ which suggests $\delta W = -\Delta U$. This is different from the work done expression in the Brayton cycle (Open cycle). In the Brayton cycle, because of the consideration of flow work the expression for work done becomes as $\delta W = -\Delta h$.

Work done in the power stroke of Otto cycle can be calculated as follows:

$$W_T = C_v * (T_3 - T_4) = 718 * (1780.46 - 774.99) = 721.927\ kJ/kg$$

Similarly, work done in the compression process is calculated below:

$$W_c = C_v * (T_1 - T_2) = 718 * (290 - 666.25) = -270.147\ kJ/kg$$

The net work produced by the Brayton cycle is as follows:

$$W_{net} = W_T + W_C = |W_T| - |W_C| = 721.927 - 270.147 = 451.779\ kJ/kg$$

Heat supplied in the constant volume combustion is as follows:

$$Q_H = \Delta U = C_v * (T_3 - T_2) = 718 * (1780.46 - 666.25) = 800.002\ kJ/kg$$

Thermal efficiency of the cycle is defined as follows:

$$\eta = \frac{|W_T| - |W_c|}{|Q_H|} * 100 = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)} = \frac{(T_3 - T_2) - (T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} \qquad [3.6]$$

Isentropic relationship is used to obtain following expressions.

$$\frac{v_1}{v_2} = \frac{v_4}{v_3}\ \&\ \frac{T_1}{T_2} = \frac{T_4}{T_3}\ \&\ \frac{P_1}{P_2} = \frac{P_4}{P_3} \qquad [3.7]$$

Using the above expressions the formula for the thermal efficiency is derived below.

$$\eta = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1 \left(\frac{T_4}{T_1} - 1\right)}{T_2 \left(\frac{T_3}{T_2} - 1\right)} = 1 - \frac{T_1}{T_2} = 1 - \frac{300}{543.43} = 1 - \frac{v_2}{v_1}^{\gamma-1} = 1 - \frac{1}{8}^{\gamma-1} = \mathbf{0.5647}\ \textbf{Ans}$$

$$\%\ \eta = 0.4479 * 100 = \mathbf{56.47}\ \% \qquad \textbf{Ans}$$

The specific volume at state 1 is as follows:

$$v_1 = \frac{R_{air} * T_1}{P_1} = \frac{287 * 290}{100000} = 0.8323 \frac{m^3}{kg}$$

$$v_2 = \frac{v_1}{8} = \frac{0.8323}{8} = 0.104 \frac{m^3}{kg}$$

The definition of the mean effective pressure is as follows:

$$P_{mean} = \frac{W_{net}}{v_{max} - v_{min}} = \frac{W_{net}}{v_1 - v_2} = \frac{451.779}{0.8323 - 0.104} = \mathbf{620.32\ kPa} \qquad \textbf{Ans}$$

Where units of the W_{net} is in J/kg and units of the specific volume v is in m^3/kg.

The mean effective pressure can also be written as follows:

$$P_{mean} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{W_{net}}{V_1 - V_2} \qquad [3.8]$$

Where units of the W_{net} is in J and units of the volume V is in m^3.

4. An ideal Diesel cycle with air as the working fluid has a compression ratio of 18 and a cut-off ratio of '2'. At the beginning of the compression process, the working fluid is at 100 kPa, 27°C, and 1917 cm3. Utilising the cold standard assumptions, determine (a) the temperature and pressure of air at the end of each process, (b) net work output, (c) thermal efficiency and (d) the mean effective pressure.

<u>Solution</u>

Given $V_1/V_2 = 18$, cut off ratio $V_3/V_2 = 2$, $T_1 = 300$ K, $P_1 = 100$ kPa and $V_1 = 1917$ cm^3.

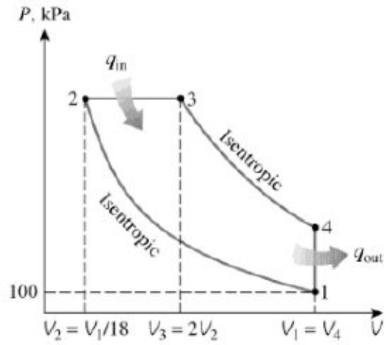


Figure 4.1

The mass of the air can be found out using the perfect gas law
$m = P_1V_1/RT_1 = 2.2265 \times 10^{-3}$ kg.
Note that $\gamma = 1.4$ for air.

(a) The process $1 - 2$ is isentropic. Therefore, $P_2 = P_1(V_1/V_2)^\gamma = 5719.81$ kPa and $T_2 = T_1(V_1/V_2)^{(\gamma-1)} = \mathbf{953.30\ K}$.
The process $2 - 3$ is isobaric. Thus, $P_3 = P_2 = \mathbf{5719.81\ kPa}$.
Also, $V_3 = 2V_2 = V_1/9 = 213$ cm^3.
Thus, $T_3 = P_3V_3/mR = \mathbf{1906.59\ K}$.

The process $3 - 4$ is isentropic and $V_4 = V_1$. Thus, using isentropic relations, we get $P_4 = P_3(V_3/V_4)^{\gamma} = 263.9$ kPa and $T_4 = T_3(V_3/V_4)^{(\gamma-1)} = \mathbf{791.7}$ **K**.

(b) Net work output $W_{net} = Q_H - Q_C = mC_P(T_3 - T_2) - mC_V(T_4 - T_1) = \mathbf{1347.49}$ **J.**
In the above equation, Q_H is the heat supplied and Q_C is the heat rejected.

(c) Thermal efficiency $\eta = W_{net}/Q_H = \mathbf{0.6316}$

(d) Mean effective pressure $= W_{net}/(V_{max} - V_{min}) = W_{net}/(V_1 - V_2) = \mathbf{744.264}$ **kPa**.

Problems to be solved in class

1.