AS 5300 - Physical Gas Dynamics
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Assignment – 1, Weightage: 10%, Total 30 Due date: Oct 01, 2023, Noon.

Assignments must be submitted through email only to ***murgi@smail.iitm.ac.in***. *In parts which require hand writing, you can write neatly in a paper and take a photo/scan, use that in a document which is typed out for remaining parts. If two manuscripts are similar,* ***both will receive negative points***. *Submitting late incurs* ***negative points of -1% per half hour***, *such that it is no point in submitting 5 hrs late or more.*

1. Consider air at 300K, 1 bar to be consisting of N_2&O_2 alone, in ratio 79:21 by volume. If we take air at 300K, 1 bar, and take it to various states given below, give the new equilibrium composition of the mixture. Consider only species N_2, O_2, NO, N, O.
(a) P=1 bar, T=2500 K; (b) P=1 bar, T=6500 K; (c) P=0.25 bar, T=2500 K; (d) P=0.25 bar, T=6500 K; (e) P= 4 bar, T=2500 K; (f) P= 4 bar, T=6500 K [6]
(g) Check if your results satisfy the Le'Chatelier principle [1]

2. Starting from Gibbs relation, derive an expression for Temperature in terms of Entropy. Use the tables for Ar at appropriate conditions, to test the values of entropy at two different temperatures. [3]

3. Consider a gas whose molecules are flat discs sliding on a frictionless plane. (Eg. Carrom coins on a frictionless board). Assume that the molecules will collide similar to billiard balls, in this plane. Assume that the gas obeys P=ρRT, where P=force per unit length of the edge of the board, ρ=density per unit area.
(a) Find a relation between P, ρ and $\overline{C^2}$. [2]
(b) Find by simple means, an expression for the mean free path. [1]
(c) Using similar arguments used by Boltzmann for deriving the velocity distribution function, find the $f(c_i)$ for these new molecules. [3]

4. A small satellite in the shape of a thin disc (D=50 cm) is travelling through atmosphere of a remote planet. The satellite's diameter D<<λ, and it travels in a direction perpendicular to the disc surface with velocity '+v'. The satellite's front surface is designed to be a sensor which can measure the collision frequency θ with which the molecules strike the satellite.
(a) If v =10 km/s, which is assumed to be far greater than the mean molecular speed in that atmosphere, and the measured value of θ= 2.5e+21/sec, find the number density of the molecules. Assume that the molecules reflect specularly from the surface of the disc. Check our D<<λ assumption, using d=3.7e-10 m. [2]
(b) Find the drag on the satellite for the above case, if the mass of molecules is 3e-26 kg. [1]
(c) If v =1 km/s, which is comparable to mean speed of molecules, estimate the number density of the molecules using more detailed analysis from advanced kinetic theory. Use T=1000K, θ= 2.5e+21/sec, m=3e-26 kg. [4]
Note: the flux of molecules can be obtained from $\Gamma=(\pi D^2/4)\theta$, and molecules have comparable movement w.r.t. disc. Error function values can be evaluated using a commercial math software like Matlab.
(d) Show that the drag coefficient on the satellite is given by: [5]

$$C_D = \frac{2}{s^2}\left\{\left(2s^2+1\right)erf(s)+\frac{2s}{\sqrt{\pi}}e^{-s^2}\right\} \text{ where } s=\frac{v}{\sqrt{2kT/m}} \text{ and } erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-z^2}dz$$

Take into account the interaction of the molecules on both the surfaces of the disc.
(e) Find the net drag in the above case. [1]

5. Consider the derivation in class for bimolecular collision rate, Z_{AB}. We used the limits 0-∞ for the dg integral. However, for reactions to occur, the relative velocity needs to have a value higher than a critical value such that $\varepsilon=\frac{1}{2}mg^2 > \varepsilon_a$. So derive the expression for the rate of effective collisions between molecules A&B. This will be the reaction rate expression used in chemical kinetics. [5]

6. **[BONUS]** Consider the same plate as above, but this time with an angle of attack. Find the lift and drag coefficients in a similar manner as 4(d). [7]