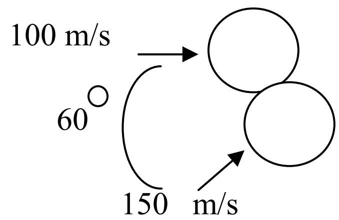
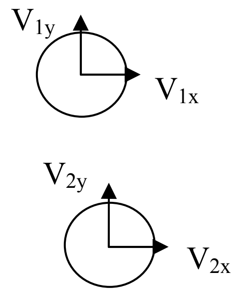
Que: 1



at the time of collision



after collision

Let's assume after the collision the velocities will be as mentioned.
Equation of conservation of linear momentum,
In x-direction: $100 * m + 150\ Cos(60) * m = V1x * m + V2x * m$ (1)
Where 'm' is the mass of each molecule.
Equation for elastic collision,

$$e = 1 = (relative\ velocity\ after\ the\ collision) / (relative\ velocity\ before\ the\ collision)$$

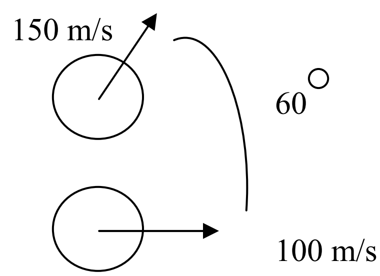
$$1 = (V1x - V2x)/(100 - 150\ Cos(60) \quad (2)$$

Momentum equation in y-direction,

$$150\ Sin(60) * m = V1y * m + V2y * m \quad (3)$$

$$1 = (V1y - V2y)/\ (0 - 150\ Sin(60) \quad (4)$$

Solving above four equations, we get the following values,
V1x = 150 Cos(60) & V1y = 150 Sin(60)
V2x = 100 & V2y = 0.
Which means the velocities will be simply exchanged along with the directions whenever we have the case of elastic collision between same size of molecules.



We can now do the similar calculations for the case of reverse collision. The magnitudes of the relative velocity will remain conserved before and after the collision for the above two cases.

Que : 2

Already done in class!!!

Que : 3

We have velocity distribution function as,

$$f(Ci) = \left(\frac{m}{2\pi kT}\right) e^{-(m/2kT)(c1^2+c2^2+c3^2)}$$

For simplicity, let's assume $\alpha = (m/2\pi T)$

$$\beta = (\frac{m}{2kT})$$

Therefore we can write, $f(Ci) = \alpha\, e^{-\beta(c1^2+c2^2+c3^2)}$

'$f(C)dc1\, dc2\, dc3$' represents the fraction of molecules having velocity vector equal to$(c1, c2, c3)$, also note that this quantity is unit less since it is a fraction. Unit of $f(Ci)$ is $sec^3/meter^3$. Now to find out the speed distribution function we have to switch the co-ordinate system to spherical polar, i.e.

$f(Ci) \equiv f(C, \Phi, \theta)$, where $C = \sqrt{C1^2} + C2^2 + C3^2$.

We will have, $f(C)dc1\ dc2\ dc3$' $\equiv f(C, \Phi, \theta)\ dC\ Cd\Phi\ CSin(\Phi)d\theta$

Both represents same fraction of molecules having velocity (C1,C2,C3) or it can also be said that speed 'C' at an azimuthal angle of 'Φ' and also at polar angle of 'θ'.

But the molecules having speed 'C' will be independent of Φ, θ therefore integrating over both angles will give us the fraction of molecules having velocity 'C'.

$$X(C)\ dC = \int_0^{\pi}\int_0^{2\pi} f(C, \Phi, \theta) dC\, Cd\Phi\, C\, Sin(\Phi) d\theta$$

First limit from 0 to pi is for azimuthal angle Φ and the second limit from 0 to 2pi is for polar angle θ.

After solving the above equation we get the following expression for speed distribution function,

$$\chi(C) = 4\boldsymbol{\pi\alpha}^{(3/2)}\boldsymbol{C}^2\boldsymbol{e}^{-(\boldsymbol{\beta C}^2)}$$

$X(C)\ dC$ represents the fraction of molecules having speed 'C' and it is a unit less quantity.

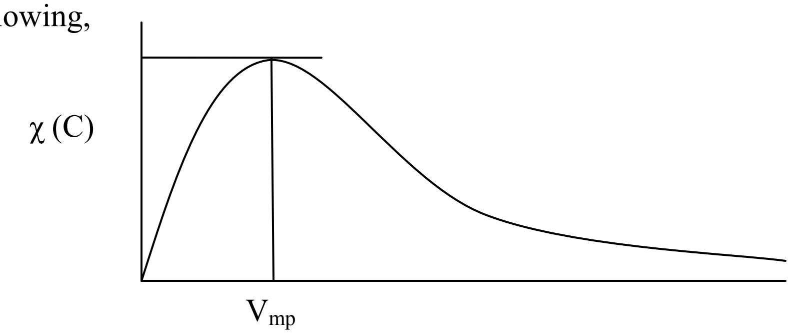
Unit of $\chi(C)$ is $(sec/meter)$, which is NOT same as unit of $f(Ci)$.

Que : 4

We have derived speed distribution function as,

$$\chi(C) = 4\boldsymbol{\pi}(\boldsymbol{m/2\pi kT})^{(3/2)}\boldsymbol{C}^2\boldsymbol{e}^{-((\boldsymbol{m/2kT})\boldsymbol{C}^2)}$$

This looks like as following,



From the above figure it is very clear that at most probable velocity we will have derivative equal to zero i.e. $d\chi/dC = 0$.
After solving we get, $Vmp = \sqrt(2kT/m)$.

The fraction of molecules having speed 'C' will be given by $'\chi(C)\, dC'$. Therefore the average speed can simply be calculated as ,$\int_0^\infty C\, \chi(C) dC$.
After solving we get, $Vavg = \sqrt(8kT/\pi m)$.

Similarly, $Vrms = \sqrt{\int_0^\infty C^2 \chi(C) dC}$
Which gives us the value equal to $\sqrt(3kT/m)$.

Que : 6

$'\chi(C)dc'$ is the fraction of molecules having speed equal to $'C'$, therefore to get the fraction of molecules having speed greater than 'V' we just have to integrate the expression from 'V' to ∞.

Required fraction, $\boldsymbol{(Rf) = \int_V^\infty \chi(C) dC.}$

$$Rf = \int_0^\infty \chi(C)dC - \int_0^V \chi(C)dC$$

$$= \int_0^\infty \alpha 1\, C^2 e^{-\beta C^2} dC - \int_0^V \alpha 1\, C^2 e^{-\beta C^2} dC,$$

where $\alpha 1 = 4\boldsymbol{\pi}(\boldsymbol{m/2\pi kT})^{(\mathbf{3/2})}$, and $\beta = m/2\boldsymbol{\pi kT}$.

$$= \left(\frac{\alpha 1}{4}\right) * \sqrt(\pi/\beta^3) - \alpha 1 * \underbrace{\int_0^V C * Ce^{-\beta C^2} dC}_{\text{I1 (say)}} \qquad (1)$$

Considering only I1, we will integrate by parts using 'C' as 1st function, we will have,

$$\text{I1} = C \int Ce^{-\beta C^2} dC - \int_0^V (\int Ce^{-\beta C^2} dC)\, dC$$

$$\text{I1} = \left[-(\frac{C}{2\beta})e^{-\beta C^2}\right] - \int_0^V -(\frac{C}{2\beta})e^{-\beta C^2} dC$$

$$\text{I1} = \left(\frac{1}{2\beta^{\frac{3}{2}}}\right) * \text{erf}\,(\sqrt{\beta} * V)$$

Putting this value of I1 in the equation (1) we will have,

$$Rf = \left(\frac{\alpha 1}{4}\right)\sqrt{\frac{\pi}{\beta^3}} \;+\; \left(\frac{\alpha 1 V}{2\beta}\right) e^{-\beta V^2} - \left(\frac{1}{2\beta^{\left(\frac{3}{2}\right)}}\right) \text{erf}\,((\sqrt{\beta})V)$$

Que : 7

We have the relation for any type of molecule, $\varepsilon = \frac{1}{2}mC^2$ (1)

Differentiating the above expression we get, $d\varepsilon = mCdC$ (2)

The molecules, having speed 'C', are having kinetic energy equal to 'ε'. Also, we know that $\chi(C)dC$ is the fraction of molecules having speed equal to $'C'$.

Now, if $\chi(\varepsilon)$ represent the kinetic energy distribution function, therefore the fraction of molecules having kinetic energy $'\varepsilon'$ will be given by $'\chi(\varepsilon)d\varepsilon'$.

Key statement: $\boldsymbol{\chi(C)dC = \chi(\varepsilon)d\varepsilon}$

We will have,

$$\chi(\varepsilon) = \chi(C)(dC/d\varepsilon)$$

Using equation (2), $\chi(\varepsilon) = \left(\frac{1}{mC}\right)\chi(C)$

$$\chi(\varepsilon) = \left(\frac{1}{mC}\right) 4\boldsymbol{\pi(m/2\pi kT)}^{(3/2)}\boldsymbol{C^2 e}^{-((m/2kT)C^2)}$$

Now putting value of 'C' in terms of 'ε' we will be having,

$$\chi(\varepsilon) = \sqrt{\frac{4\varepsilon}{\pi(kT)^3}}\, e^{-(\varepsilon/kT)}$$
