Que : 1

a.)

The total number of macro states are listed below,

ε1	ε2	ε3	ε4	ε5
1	1	1	1	1
0	2	1	2	0
2	0	1	2	0
1	1	0	3	0
1	0	3	0	1

But all the macro states will not be possible because of given degeneracy levels. Therefore only possible macro states will be,

ε1 (g = 1)	ε2 (g = 1)	ε3 (g = 3)	ε4 (g = 4)	ε5 (g = 4)
1	1	1	1	1
1	1	0	3	0
1	0	3	0	1

For the 1st macro state the total number of microstates will be multiplication of microstates corresponding to the each energy state, which is as following,

$$\prod_j \left(\frac{gj!}{(gj - cj)!\,Nj!}\right)$$

1×1×3×4×4 =48.

Similarly for the 2nd macro state we will have number of micro states equal to 1×1×4 = 4.

And for 3rd macro state we will have number of micro states equal to 1×1×4 = 4.

The molecules try to have distribution (a macro state) in such a way so that they can have maximum number of microstates.

Distribution for N=5 and E=20 is not possible for Fermi-Dirac statistic. And for N= 10 & E=20 solution will be given later.

<u>NOTE:</u>

- If we have E fixed and have increment in the number of molecules then the molecules will try to occupy the lower energy states as before in order to satisfy the total energy conservation criteria.
- But if the total number of molecules are fixed and we have increment in the total energy then the molecules will occupy the higher energy states in order to full fill the new higher energy criteria.

Que : 2

The possible macro states will be as following,

ε1 (g = 2)	**ε2 (g = 2)**	**ε3 (g = 2)**	**ε4 (g = 2)**	**ε5 (g = 2)**
1	1	1	1	1
0	2	1	2	0
2	0	1	2	0

For the 1st macro state the total number of microstates will be = $2\times2\times2\times2\times2 = 32$.

Similarly for the 2nd macro state the total number of micro states will be = $1\times2\times1 = 2$.

And for 3rd macro state the total number of micro states will be = $1\times2\times1 = 2$.

In this case the, at equilibrium the 1st macro state will be the governing state because it has maximum number of microstates.

<u>NOTE:</u>

In this question we have same N, E and energy states (ε_i) as in question 1 (a) but the only difference in g_i (degeneracy). The 1st macro state is same in both the cases but we have difference in microstates because of degeneracy.

- Therefore if degeneracy is more in higher energy states molecules will have much more microstates for a given macro state.

Que : 3

In case of Boson, there can be any number of particles on each energy state i.e. this model doesn't care about degeneracy.

Lets pick the case, N = 5 & E = 10.

All the possible macro states will be as following,

ε1	**ε2**	**ε3**	**ε4**	**ε5**
1	1	1	1	1
0	2	1	2	0
2	0	1	2	0
1	1	0	3	0
1	0	3	0	1

For a given macro state, we have the formula to calculate the total number of micro state in Boson model,

$$\prod_{j} \frac{(Nj + gj - 1)!}{Nj!\,(gj - 1)!}$$

So for 1st macro state, the total number of micro states will be = $1 \times 1 \times 3 \times 4 \times 4$ = 48.

Similarly, for 2nd macro state, the total number of micro states will be = $1 \times 1 \times 10$ = 10.

For 3rd macro state, the total number of micro states will be = $1 \times 3 \times 10 = 30$.

For 4th macro state, the total number of micro states will be = $1 \times 1 \times 20$ = 20.

For 5th macro state, the total number of micro states will be = $1 \times 20 \times 1$ = 20.

The dominating macro state will be the first one i.e. at equilibrium the molecules will have distribution as 1st macro state.

NOTE: In the above three questions we can see that there is one particular macro state which has maximum number of microstates than the other macro states. Which means there is only one macro state which has maximum contribution to the microstates as compared to other macro states.

Que : 4

Fermion case,

(here Cj notation has been used in place of gj)

For a given macro state, total number of microstates possible are given below,

$$W(N1, N2, \ldots) = \Pi \frac{Cj!}{(Cj - Nj)!Nj!} \qquad (1)$$

Now to obtain total number of microstates for all macro states, we will have to add up all the microstates for corresponding macro state.

$$\Omega = \sum W\,(N1, N2, \ldots .)$$

Provided, $\sum_j Nj = N$ and also, $\sum_j \varepsilon jNj = E.$

No we make the approximation that the contribution from all macro states other than the one corresponding to the largest number of microstates is negligible. i.e. we are going to choose the one particular distribution of N1, N2, N3, … in which we will be having maximum number of micro states and that distribution (or that macro state) will have largest contribution to **Ω**, other contributions can be ignored.

$$\Omega \cong Wmax$$

We will write Nj as Nj* for this particular macro state. And if W corresponding to that macro state is maximum then ln(W) will also be maximum.

Taking log on both sides in the above equation (1),

$$lnW = \sum_j \ln Cj! - \ln(Cj - Nj)\,! - \ln Nj! \qquad (2)$$

Now remember, $\ln Z! \cong Z \ln Z - Z$ (Stirling's formula)

$$\ln W \cong \sum_j [-Cj \ln\left(1 - \frac{Nj}{Cj}\right) + Nj \ln\,(\frac{Cj}{Nj} - 1)] \qquad (3)$$

Note that Cj >> 1, for maximization $d(\ln W) = 0$

Here we assume that the function $\ln W$ behaves as a continuous function of Nj subject to the conditions $\sum_j Nj = N$ and $\sum_j \varepsilon jNj = E.$

We will adopt the method of Lagrange multiplier (G), i.e.

$$G = \ln W - \alpha\left(\sum_j Nj - N\right) + \beta\left(\sum_j \varepsilon jNj - E\right)$$

dG = 0 for max(lnW), i.e.

$$dG = \frac{\partial(lnW)}{\partial Nj}\ dNj - \alpha\ dNj\ +\ \beta\ \varepsilon j\ dNj = 0$$

Since Cj is fixed by us and only Nj is varying.

$$\left\{\ln(\frac{Cj}{Nj} - 1) - \alpha -\ \beta\ \varepsilon j\right\} = 0$$

$$\ln(\frac{Cj}{Nj} - 1) =\ \alpha +\ \beta\ \varepsilon j$$

$$\frac{Nj *}{Cj} =\ \frac{1}{e^{(\alpha+\beta\varepsilon j)} + 1}$$

Nj* is distribution corresponding to $Wmax$.
