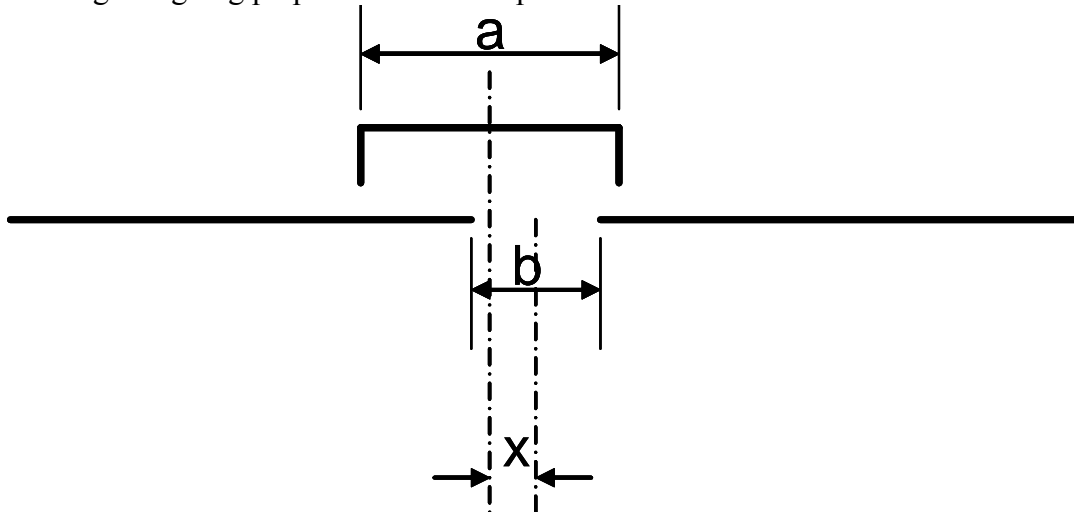


AS – 6342 Spectroscopic Reactive Flow Diagnostics
Dr. T. M. Muruganandam
Supplementary Exercise – 3
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Convolution: Convolution of two functions f & g is defined as $(f \otimes g)(x) = \int_{-\infty}^{+\infty} f(x-y)g(y)dy$

1. Prove that $\int_{-\infty}^{+\infty} f(x-y)g(y)dy = \int_{-\infty}^{+\infty} f(y)g(x-y)dy = (f \otimes g)(x) = (g \otimes f)(x)$

2. There is a detector of width 'a' which is placed behind a slot of width 'b', as illustrated in the schematic. The light can reach the detector only through the slot and from no other direction. Also light is going perpendicular to the planes of the detector and the slot.



Plot the intensity of light expected as a function of the location 'x' of the slot-center, w.r.t. the detector center. Use values of x to be $(-\infty, +\infty)$.

3. Rect function is a function defined by $\Pi(x) = \begin{cases} 1 & \text{for } -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$.

- (a) Find the function $\Pi(x/b)$.
- (b) Find the function form of $\Pi(x/a) \otimes \Pi(x/b)$, Also plot it!
- (c) Is it the same as $\Pi(x/b) \otimes \Pi(x/a)$?
- (d) How is this problem related to the problem 2? Prove it.

4. Imagine a gradient like slot in place of that in problem 2. That is, at the left end of the slot, there will be no light entering, and the right end full light entering, and at middle of the slot, 50% of light entering the detector. Find the response of the detector for this slot moving from left to right of the detector.

Use ramp function defined as $Ramp(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

5. Find the following functions: (also plot them)

(a) $\Pi(x/2) \otimes \Pi(2x)$

(b) $\Pi((x-1)/2) \otimes \Pi(2(x-3))$

(c) $\text{Ramp}(x/2) \otimes \Pi(x/3)$

(d) $-\text{Ramp}(x) \otimes \text{Ramp}(-x)$

(e) $-\text{Ramp}(x) \otimes -\Pi(-x)$

(f) $\delta(x) \otimes f(x)$, where $\delta(x)=1$ for $x=0$, and zero everywhere else. $\delta(x)$ is an impulse function.

6. Find $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \otimes \frac{d/\pi}{d^2 + x^2}$ for various values of σ and d (choose values between 0 and 2).

Use large number of points and integrate numerically instead of trying direct integration of this function (which is very complicated!).

The function on the left is the Gaussian distribution function, and the function on the right is the Lorentzian distribution function. The profiles you have generated are called the Voigt Profiles! They are very much used in spectroscopy. Do some web research on that!

7. Use MATLAB or equivalent softwares to get FFT of the signals in problem 5, multiply the functions in the FFT space, and then invert the FFT. Show that the results are the respective convolutions.