

AS2070: Aerospace Structural Mechanics Module 2: Composite Material Mechanics

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Chapters 1-3 in Gibson (2012).



MECHANICS OF



Chapter 25 in Megson (2013)

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Introduction

- Structural material consisting of multiple non-soluble macro-constituents.
- Main motivation: material properties tailored to applications.
- Both stiffness and strength comes from the fibers/particles, and the matrix holdes everything together.



Types of composite materials (Figure from NPTEL Online-IIT KANPUR (2025))

Examples

- Reinforced concrete
- Wood (lignin matrix reinforced by cellulose fibers)
- Carbon-Fiber Reinforced Plastics (CFRP)

Introduction

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Continuous fibres

Short fibres/Whiskers

Particulate

Types of composite materials (Figure from NPTEL Online-IIT KANPUR (2025))

Examples

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Introduction

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Introduction



1.2. Modeling Composite Material

Introduction

Two main approaches:



1.2. Modeling Composite Material

Introduction

Two main approaches:



Introduction



• Strain is fixed, but stress experienced by media differ.

 $\sigma_l = E_l \varepsilon_l$

• Stress-strain relationship simplifies as,

$$\sigma_m = E_m \varepsilon_l, \quad \sigma_f = E_f \varepsilon_l$$
$$\sigma_l A = \sigma_m A_m + \sigma_f A_f$$
$$\implies \boxed{E_l = \frac{A_f}{A} E_f + \frac{A_m}{A} E_m}.$$

Introduction



• Strain is fixed, but stress experienced by media differ.

 $\sigma_l = E_l \varepsilon_l$

• Stress-strain relationship simplifies as,

$$\begin{split} \sigma_m &= E_m \varepsilon_l, \quad \sigma_f = E_f \varepsilon_l \\ \sigma_l A &= \sigma_m A_m + \sigma_f A_f \\ \implies \boxed{E_l &= \frac{A_f}{A} E_f + \frac{A_m}{A} E_m}. \end{split}$$

Transverse Elongation



• Stress is fixed, strains differ:

$$\begin{split} \varepsilon_t l_t &= \varepsilon_m l_m + \varepsilon_f l_f \\ \Longrightarrow \; \frac{\sigma_t}{E_t} l_t &= \frac{\sigma_t}{E_m} l_m + \frac{\sigma_t}{E_f} l_f \\ \Longrightarrow & \boxed{\frac{1}{E_t} = \frac{1}{E_m} \frac{l_m}{l_t} + \frac{1}{E_f} \frac{l_f}{l_t}}. \end{split}$$

(Figures fro	om Megson	2013)
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Introduction: Poisson Effects



(Figures from Megson 2013)

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Introduction: Poisson Effects



(Figures from Megson 2013)

Introduction: Poisson Effects



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Introduction: Anisotropy

General Anisotropy

σ_{xx}		C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	ε_{xx}
σ_{yy}		C_{12}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}	ε_{yy}
σ_{zz}	_	C_{13}	C_{23}	C_{33}	C_{34}	C_{35}	C_{36}	ε_{zz}
σ_{xy}	=	C_{14}	C_{24}	C_{34}	C_{44}	C_{45}	C_{46}	γ_{xy}
σ_{xz}		C_{15}	C_{25}	C_{35}	C_{45}	C_{55}	C_{56}	γ_{xz}
σ_{yz}		C_{16}	C_{26}	C_{36}	C_{46}	C_{56}	C_{66}	γ_{yz}

Introduction: Anisotropy

General Anisotropy

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

Monoclinic: Single Plane of Symmetry

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & 0 & 0 \\ C_{12} & C_{22} & C_{23} & C_{24} & 0 & 0 \\ C_{13} & C_{23} & C_{33} & C_{34} & 0 & 0 \\ C_{14} & C_{24} & C_{34} & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & C_{56} \\ 0 & 0 & 0 & 0 & C_{56} & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

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Introduction: Anisotropy



Transversely Isotropic

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{xz} \\ \sigma_{yz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11} - C_{12}}{2} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$$

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1.4. Classical Laminate Theory

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2. Composite Materials



Griffith's experiments with glass fibres (1920) (Figure from Gibson 2012)

2. Composite Materials



2.1. Types of Composite Materials

Composite Materials



FIGURE 1.4

Types of fiber-reinforced composites. (a) Continuous fiber composite, (b) woven composite, (c) chopped fiber composite, and (d) hybrid composite.

(Figure from Gibson 2012)

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