



AS2070: Aerospace Structural Mechanics

Module 3: Introduction to Fatigue and Failure

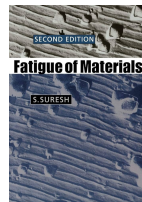
Instructor: Nidish Narayanaa Balaji

Department of Aerospace Engineering, IIT Madras

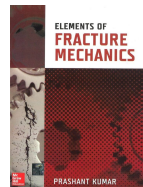
April 8, 2025

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 - The deHavilland Comet



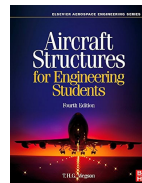
Chapters 1,7,9
in Suresh (1998)



Chapters 1-3
in Kumar (2009)



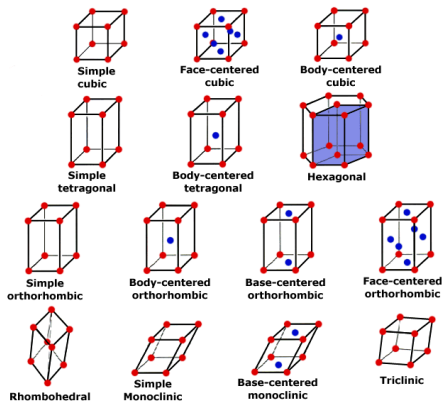
Chapter 3 in Jr
and Rethwisch
(2012)



Chapter 15
in Megson (2013)

1.1. Structure of Materials

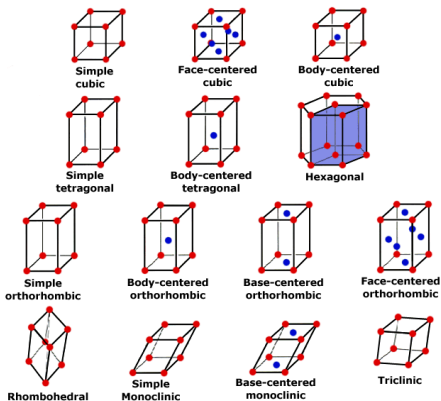
Introduction



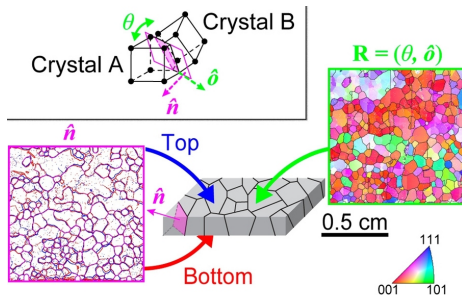
*Types of crystal structures in metals Sparky
(2013)*

1.1. Structure of Materials

Introduction



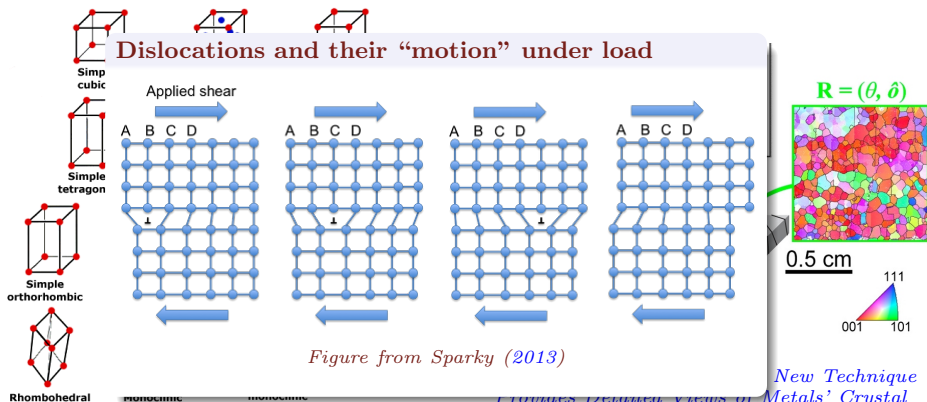
Types of crystal structures in metals Sparky (2013)



Crystal and Grain Structures New Technique Provides Detailed Views of Metals' Crystal Structure (2016). "Polycrystallinity"

1.1. Structure of Materials

Introduction



Types of crystal structures in metals Sparky (2013)

New Technique Provides Detailed Views of Metals' Crystal Structure (2016). “Polycrystallinity”

1.2. Understanding the Stress-Strain Curve

Introduction

The Uniaxial Tensile Test

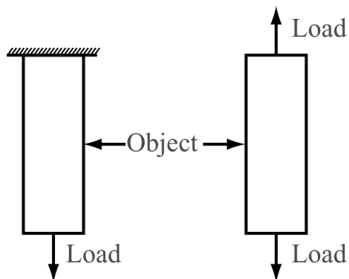


Figure from Rajendran 2011

1.2. Understanding the Stress-Strain Curve

Introduction

Terminology

- ➊ Proportionality Limit;
- ➋ Elastic Limit;
- ➌ Yield Point;
- ➍ Ultimate Strength;
- ➎ Fracture Point;
- ➏ Elongation at Failure;

Ductile Fracture

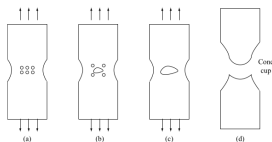


Figure from Rajendran
2011

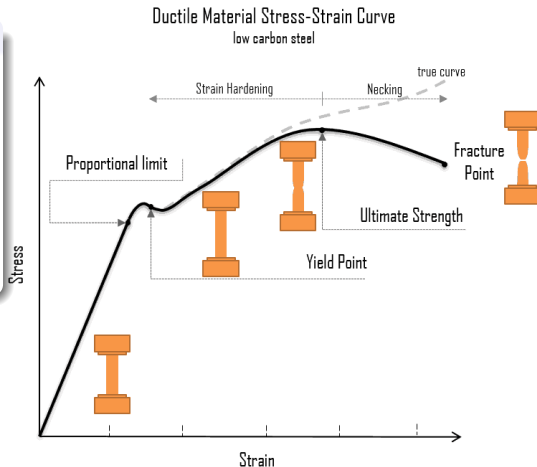


Figure from Connor 2020

1.3. Failure Mechanisms: Fracture

1. Introduction

“Griffith Theory” of brittle fracture

- Theoretical fracture stress
 $\sim \frac{E}{5} - \frac{E}{30}$ (steel $\sim \frac{E}{1000}$)

- Fracture occurs when
 $E_{strain} = E_{surface}$

- Crack propagates when
 $\frac{dE_{strain}}{dL} = \frac{dE_{surface}}{dL}$

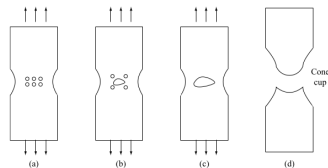
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Ductile Fracture



Ductile Fracture Rajendran 2011

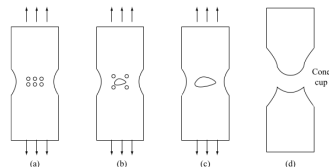
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Ductile Fracture



Ductile Fracture Rajendran 2011

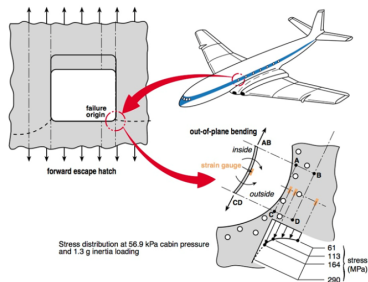
Sr. No	Brittle Fracture	Ductile Fracture
1.	It occurs with no or little plastic deformation.	It occurs with large plastic deformation.
2.	The rate of propagation of the crack is fast.	The rate of propagation of the crack is slow.
3.	It occurs suddenly without any warning.	It occurs slowly.
4.	The fractured surface is flat.	The fractured surface has rough contour and the shape is similar to cup and cone arrangement.
5.	The fractured surface appears shiny.	The fractured surface is dull when viewed with naked eye and the surface has dimpled appearance when viewed with scanning electron microscope.
6.	It occurs where micro crack is larger.	It occurs in localised region where the deformation is larger.

Ductile vs Brittle Fracture Rajendran 2011

1.3. Failure Mechanisms: Fatigue

1. Introduction

..over 90% of mechanical failures are caused because of metal fatigue *What Is Metal Fatigue?* 2021...

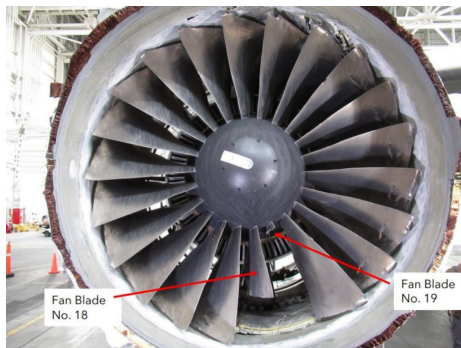


The De Havilland Comet The deHavilland Comet Disaster 2019 [lecture]

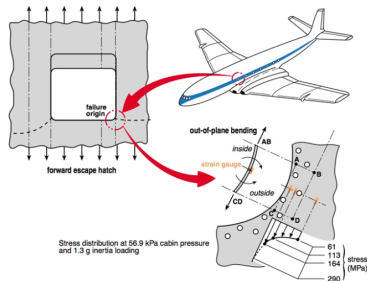
1.3. Failure Mechanisms: Fatigue

1. Introduction

..over 90% of mechanical failures are caused because of metal fatigue *What Is Metal Fatigue? 2021...*



A more recent example (2021 United Airlines Boeing 777) **DCA21FA085Aspx.** [\[video\]](#)



The De Havilland Comet *The deHavilland Comet Disaster 2019* [\[lecture\]](#)

1.3. Failure Mechanisms: Fatigue

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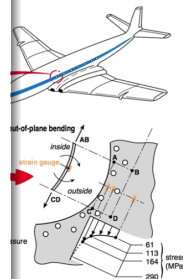
Fatigue Crack Propagation: Beech Marks



A more recent exam
Boeing 777) DCA



Figure from Fatigue Physics 2024

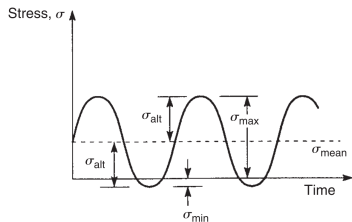


net The deHavilland
2019 [lecture]

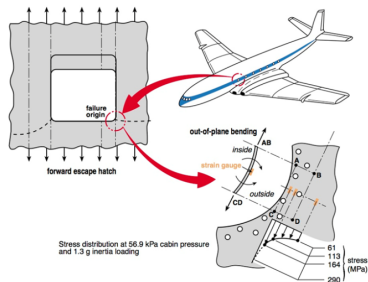
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Fatigue variables Megson 2013

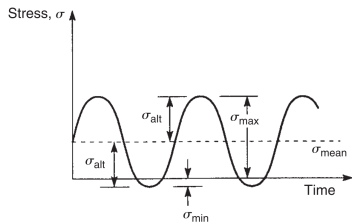


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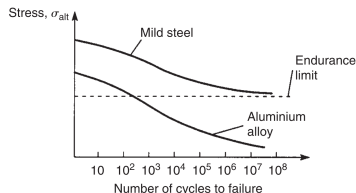
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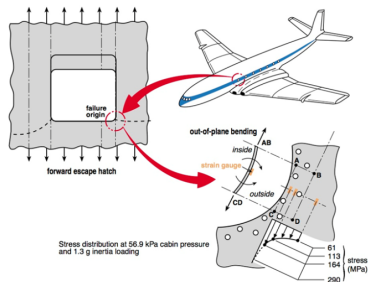
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Fatigue variables Megson 2013



The S-n Diagram Megson 2013



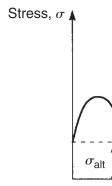
The De Havilland Comet The deHavilland Comet Disaster 2019 [lecture]

1.3. Failure Mechanisms: Fatigue

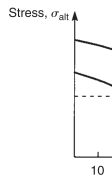
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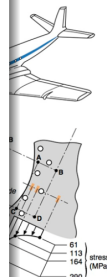
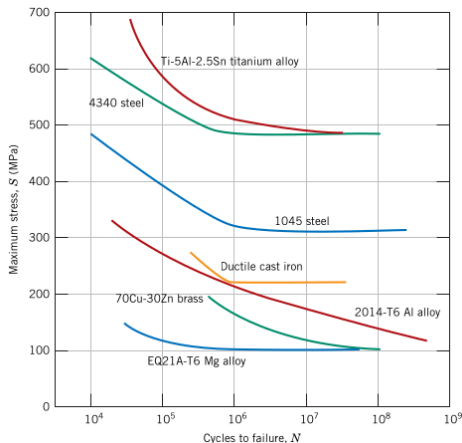
S-N Curves for Common Metals (Jr and Rethwisch 2012)



Fatigue



The S-n



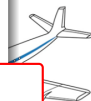
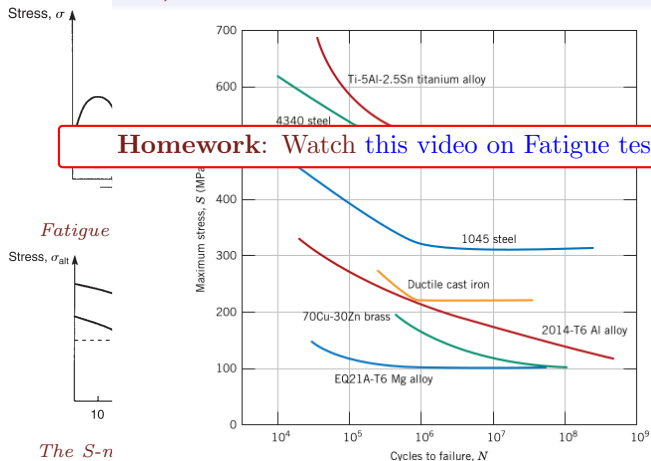
*deHavilland
ecture]*

1.3. Failure Mechanisms: Fatigue

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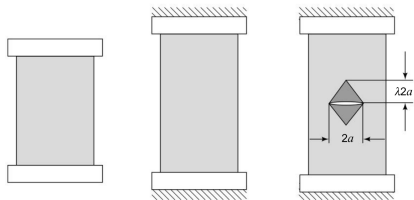
S-N Curves for Common Metals (Jr and Rethwisch 2012)



*deHavilland
ecture]*

1.4. Energy Release Rate: Griffith's Analysis

Introduction



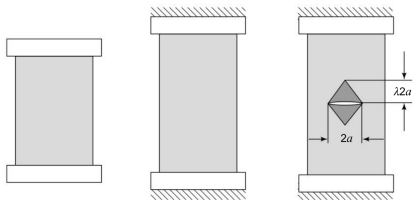
Simplistic picture of the introduction of a crack in a stretched specimen (Figure from sec 2.5 in Kumar 2009)

- Because of the crack, we assume $\sigma \approx 0$ in the triangles.
- Corresponding energy loss:

$$E_R = V_{\Delta} \times \left(\frac{\sigma^2}{2E} \right) = \frac{2a^2 \lambda t \sigma^2}{E}.$$

1.4. Energy Release Rate: Griffith's Analysis

Introduction



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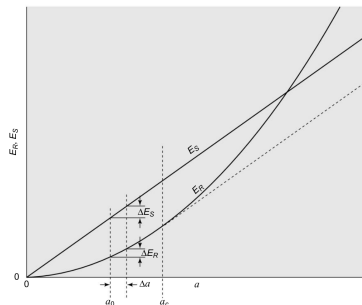
$$E_R = V_{\Delta} \times \left(\frac{\sigma^2}{2E} \right) = \frac{2a^2 \lambda t \sigma^2}{E}.$$

- For thin plates, $\lambda = \frac{\pi}{2}$. So,

$$E_R = \frac{\pi a^2 t \sigma^2}{E}.$$

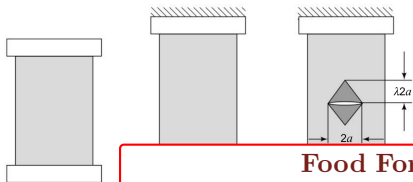
- The “creation” of a surface takes energy. We write this as,

$$E_S = 2(2at)\gamma = 4at\gamma.$$



1.4. Energy Release Rate: Griffith's Analysis

Introduction



*Simplistic picture
in a stretched
(2009)*

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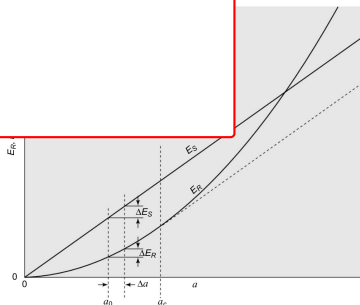
Food For Thought

- What would a “safe size” of crack be, for a given loading condition? *Hint: Think incrementally*

- Because $\sigma \approx 0$ in

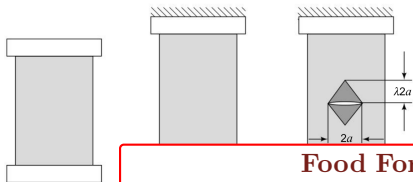
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1.4. Energy Release Rate: Griffith's Analysis

Introduction



*Simplistic picture
in a stretched
(2009)*

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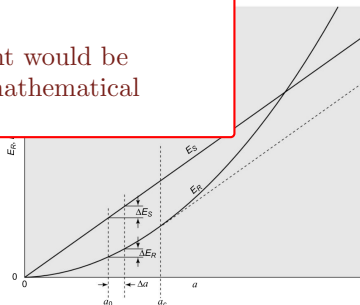
- For thin plates, $\lambda = \frac{\pi}{2}$. So,

$$E_R = \frac{\pi a^2 t \sigma^2}{E}.$$

- The “creation” of a surface takes this as, $\gamma = 4at\gamma$.

Food For Thought

- What would a “safe size” of crack be, for a given loading condition? *Hint: Think incrementally*
- What type of an experiment would be necessary to confirm this mathematical framework?



1.5. Linear Elastic Fracture Mechanics

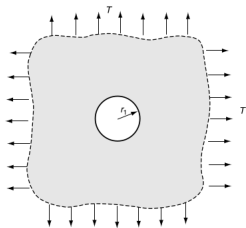
Introduction

(Ref: Sec. 8.4.2 in Sadd 2009)

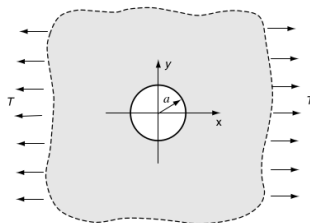
Consider the following two cases.

Question: Where will the point of peak stress occur? And which will have higher maximum stress?

Case 1



Case 2



1.5. Linear Elastic Fracture Mechanics

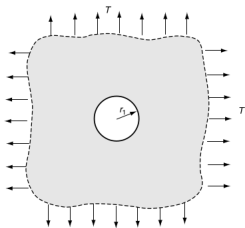
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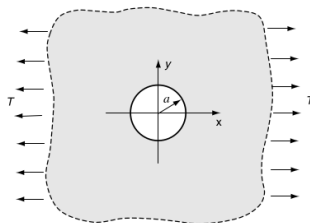
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Case 2



Analytical Solution

$$\sigma_r = T(1 - \frac{r_1^2}{r^2}), \sigma_\theta = T(1 + \frac{r_1^2}{r^2})$$

$$\Rightarrow \sigma_{\max} = 2T$$

1.5. Linear Elastic Fracture Mechanics

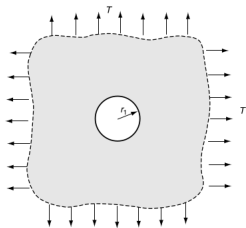
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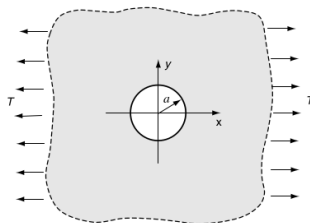
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Analytical Solution

$$\sigma_r = T(1 - \frac{r_1^2}{r^2}) + (\cdot) \cos(2\theta), \sigma_\theta = \dots$$

$$\Rightarrow \sigma_{\max} = 3T$$

1.5. Linear Elastic Fracture Mechanics

Introduction

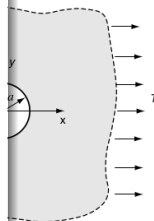
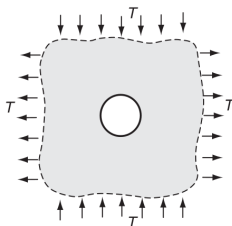
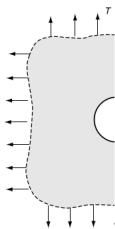
(Ref: Sec. 8.4.2 in Sadd 2009)

Consider the following two cases.

Question: Where will the point of peak stress occur? And which will have higher maximum stress?

Case 3

Case 1



Analytical Solution

$$\sigma_{\max} = 4T$$

$$\sigma_r = T\left(1 - \frac{r_1^2}{r^2}\right), \sigma_\theta = T\left(1 + \frac{r_1^2}{r^2}\right)$$

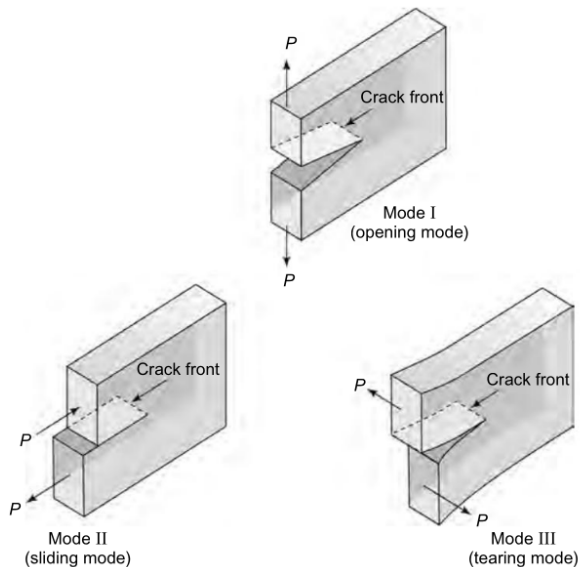
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1.6. Modes of Fracture

Introduction

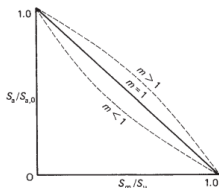


2. Introduction to Fatigue

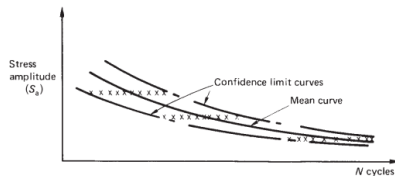
Concepts

- Safe Life: RUL
- Fail-Safe: Redundancy

Tensile Stresses: The Goodman Diagram

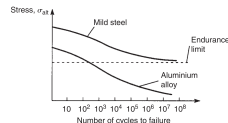


(Figure 15.2 from Megson 2013)



(Figure 15.1 from Megson 2013)

The S-N Curve



(Figure from Megson 2013)

$$\sigma_{alt} = \sigma_{\infty} \left(1 + \frac{C}{\sqrt{N}} \right), \quad N \propto \frac{1}{\sigma_{mean}^2}$$

2.1. The deHavilland Comet

Introduction to Fatigue

No aircraft has contributed more to safety in the jet age than the Comet. The lessons it taught the world of aeronautics live in every jet airliner flying today. – D.D. Dempster, 1959, in The Tale of the Comet

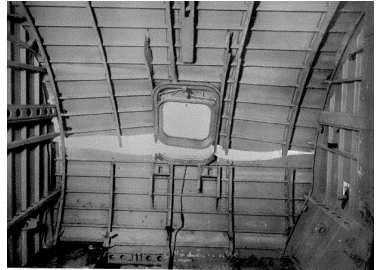


FIG. 7. VIEW FROM INSIDE OF FAILURE AT THE FORWARD ESCAPE HATCH ON THE PORT SIDE—COMET G-ALYU

(Figures from “De Havilland Comet” 2025)

2.1. The deHavilland Comet

Introduction to Fatigue

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The Tale of the Comet

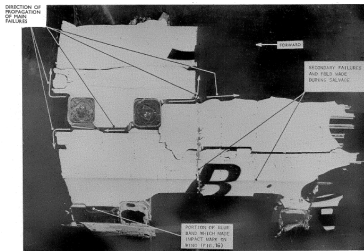


FIG. 12. PHOTOGRAPH OF WRECKAGE AROUND ADF AERIAL WINDOWS—G-ALYP.

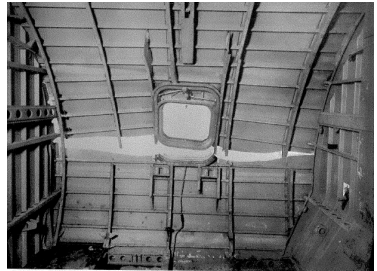


FIG. 7. VIEW FROM INSIDE OF FAILURE AT THE FORWARD ESCAPE HATCH ON THE PORT SIDE—COMET G-ALYU

(Figures from “De Havilland Comet” 2025)

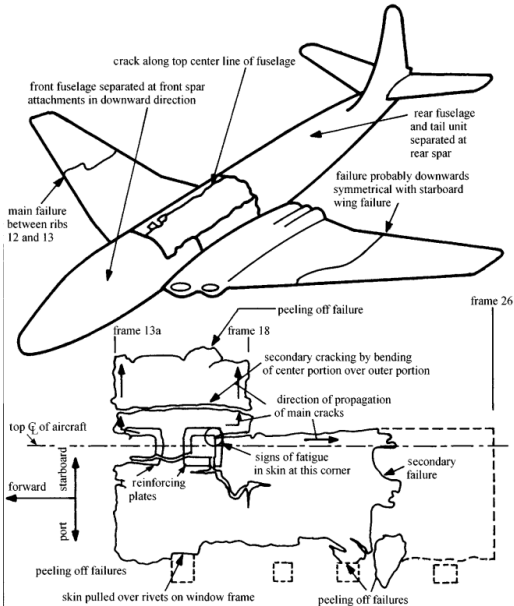
2.1. The

Introduction to

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FIG. 12. PHOTO



Comet.
r flying



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t" 2025)

References I

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