Box Beam Sizing Tutorial

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§1. Problem Setup

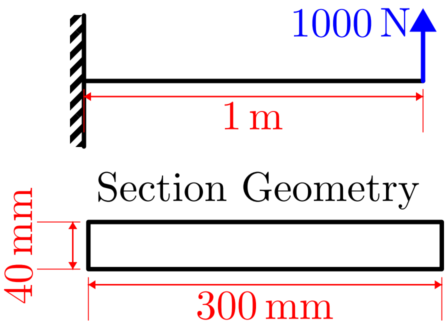

Figure 1: Problem Setup

Let us consider the cantilevered beam shown in Figure 1, and take everything that is given there as "specification". Let us also take for granted that the material properties are also given:

$$E_y = 70 \text{ GPa}, \quad \nu = 0.3, \quad S_y = 500 \text{ MPa}.$$

Let the desired factor of safety by 3 for the design. Our task will be to synthesize such a beam that can safely take the specified load while maintaining the section geometry without collapse.

§2. Sizing Calculations

We shall attempt to make the section thin walled to minimize the weight of the beam. The main two considerations involved for our calculations are:

1. Material Failure (yielding)
2. Component Buckling

For both of these, we shall estimate an allowable stress as the failure/critical stress divided by the factor of safety (taken as 3 here) and size the individual components so that their stresses do not exceed this.

§2.1. Rib Design

If we were to simply apply a point load somewhere on the section at the tip, it's going to collapse and we will violate the specifications (see Figure 2). We shall now add a "rib" at the tip in the form of a plate with uniform thickness. We need to determine the thickness of this plate.

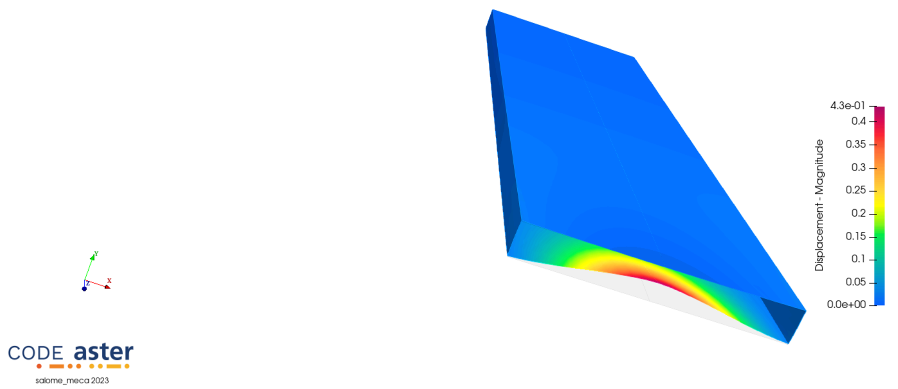

Figure 2: Collapse of the tip section under point load (displacement visualization scaled by 0.05x)

Let us assume that the load is acting at the centroid of the rib. This will be supported by the shear stresses on the walls of the beam. Although it is possible to develop better approximations of the shear stresses, we will take a simplified approach and have the webs supporting the rib symmetrically.

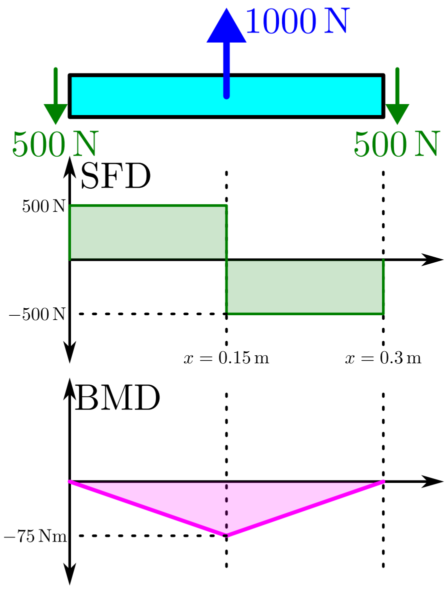

Figure 3: FBD of Rib

Figure 3 shows the shear force and bending moment diagrams drawn for the rib section. Let us first consider the shear force - it can be seen that the maximum shear force is 500 N, which will lead to a shear stress level of $\frac{500}{0.04 t_{\text{rib}}}$, where t_{rib} is the (yet to be determined) web thickness of the rib. So the shear stress in the rib is:

$$\tau_{\text{rib}} = \frac{500}{0.04 t_{\text{rib}}} = \frac{12500}{t_{\text{rib}}}.$$

Considering the bending moment diagram, we can write down the axial stress as:

$$\sigma_{\text{rib}} = -500x \times \frac{0.02}{I_{\text{rib}}} = -\frac{10x}{I_{\text{rib}}}.$$

The sizing exercise here is to find the required thickness t_{rib} and second area moment I_{rib}. Note that the "web" portion of the rib alone will have second area moment $t_{\text{rib}} \frac{0.04^3}{12}$ and if this turns out to be lesser than the required I_{rib}, then we will have to add a "flange" to the rib to reinforce its ends.

The 2D state of stress (we assume that the rib plate will be thin enough to justify plane stress assumptions) at any point along the chord (larger, horizontal dimension) of the rib can, hereby be written as

$$\underset{\sim}{\sigma} = \begin{pmatrix} -\frac{10x}{I_{\text{req}}} & \frac{12500}{t_{\text{rib}}} \\ \frac{12500}{t_{\text{rib}}} & 0 \end{pmatrix}.$$

The **Von Mises Stress** corresponding to this state of stress comes out to be:

$$\sigma_{\text{vm}} = \frac{10}{t_{\text{rib}} I_{\text{rib}}} \sqrt{4687500 I_{\text{rib}}^2 + t_{\text{rib}}^2 x^2}.$$

To avoid material failure, $\sigma_{\text{vm}} \leq \frac{S_y}{3}$, i.e.,

$$\frac{\sqrt{4687500 I_{\text{rib}}^2 + t_{\text{rib}}^2 x^2}}{t_{\text{rib}} I_{\text{req}}} \leq 1.67 \times 10^7.$$

Coming to buckling criteria, the critical stresses can be written as:

$$\sigma_{\text{crit}} = 4\frac{\pi^2 E_y}{12(1-\nu^2)} \left(\frac{t_{\text{rib}}}{b}\right)^2, \qquad \tau_{\text{crit}} = 5.35 \frac{\pi^2 E_y}{12(1-\nu^2)} \left(\frac{t_{\text{rib}}}{b}\right)^2,$$

where b is the smallest base dimension in each case. For compression, only half the rib is under compression (so $b = 20$ mm), and we will take the mean stress to be half of the peak σ. For shear we will have $b = 40$ mm since the whole section is under shear. After some manipulations, we get

$$\frac{5x}{I_{\text{rib}}} \leq 2.2048 \times 10^{14} t_{\text{rib}}^2, \qquad \frac{12500}{t_{\text{rib}}} \leq 7.0516 \times 10^{13} t_{\text{rib}}^2$$

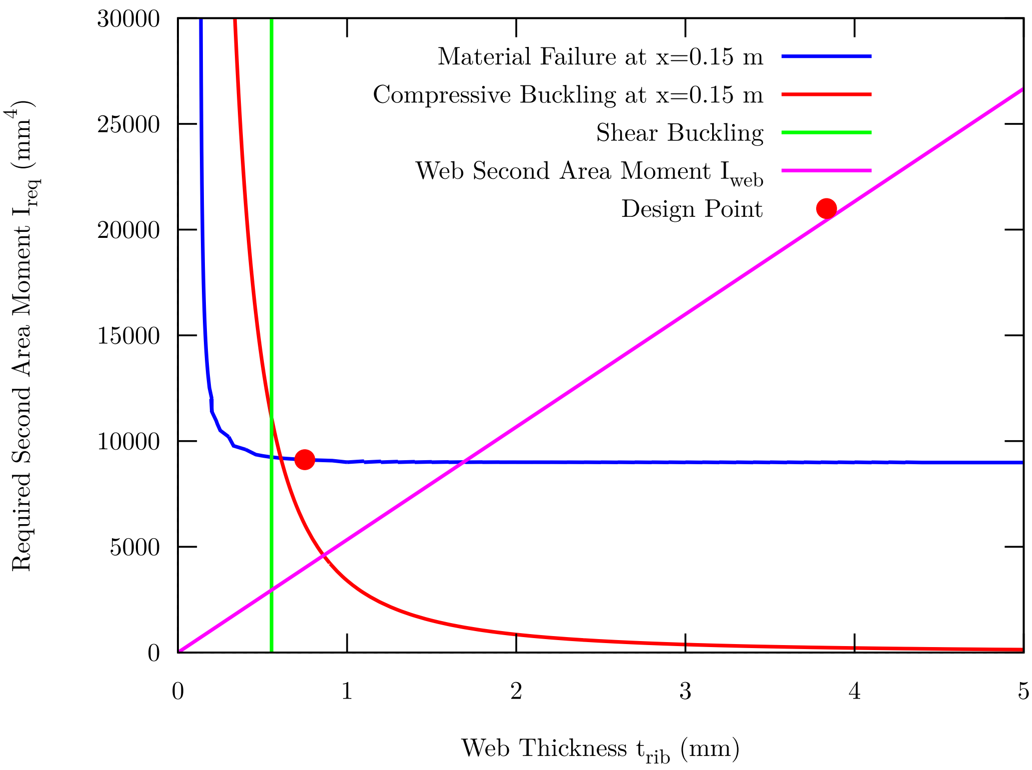

Figure 4: The Rib Sizing Curves - the first 3 curves specify lower bounds for the parameters. The last curve shows the contribution of the web thickness - flanges need to be added if $I_{\text{req}} > I_{\text{web}}$. The design point corresponds to $t_{\text{rib}} = 0.75$ mm, $I_{\text{req}} = 9119\text{mm}^4$.

Figure 4 depicts the sizing curves for the rib. The three constraints above (material failure from Von Mises, compressive buckling, and shear buckling) are plotted along with the second area moment contribution from the web alone: $I_{\text{web}} = \frac{t_{\text{rib}} 0.04^3}{12}$.

From this we fix the **thickness of the rib web** $t_{\text{rib}} = 0.75$ **mm** and **required second area moment** $I_{\text{req}} = 9119\text{mm}^4$. The web itself offers $I_{\text{web}} = 4000\text{mm}^4$ and we need a flange to contribute $I_{\text{flange}} = I_{\text{req}} - I_{\text{web}} = 5119\text{mm}^4$.

Our preliminary design of the flange will be just two concentrated areas at the top and bottom of the rib. For $I_{\text{flange}} = 5119\text{mm}^4 = A_{\text{flange}} \times 20^2 \times 2$, we get $A_{\text{flange}} = 6.4\text{mm}^2$.

This concludes the rib design.

§2.2. Skin Sizing

Now let us size the skin. Considering the webs, which carry shear stresses of $\frac{500}{0.04 t_{\text{web}}} = \frac{12500}{t_{\text{web}}}$, applying the material failure criterion here leads to

$$\frac{12500}{t_{\text{web}}} \leq \frac{500 \times 10^6}{3\sqrt{3}} \Rightarrow t_{\text{web}} \geq 0.13 \text{ mm}.$$

Since the web length itself is just 40 mm, we would like to avoid having to reinforce it for shear buckling. So let's find the required web thickness to protect against shear buckling:

$$\frac{12500}{t_{\text{web}}} \leq \frac{5.35}{3} \frac{\pi^2 E_y}{12(1-\nu^2)} \left(\frac{t_{\text{web}}}{0.04}\right)^2 \Rightarrow t_{\text{web}} \geq 0.562 \text{ mm}.$$

Let's choose the web thickness to be 0.6 mm.

For the flanges, we consider the axial stresses, since this will dominate here. The peak stress will be at the root, with second area moment 1000 Nm. The material failure due to axial stress criterion leads to:

$$\frac{1000 \times 0.02}{I_{\text{req}}} \leq \frac{500 \times 10^6}{3} \Rightarrow I_{\text{req}} \geq 1.2 \times 10^{-7} \text{m}^4.$$

Assuming a flange thickness of t_{flange}, the section second area moment gets written as

$$I_{\text{sec}} = 0.3 t_{\text{flange}} \times 2 \times 0.02^2 + 2 \times \frac{t_{\text{web}} \times 0.04^3}{12} = 2.4 \times 10^{-4} t_{\text{flange}} + 6.4 \times 10^{-9} \text{mm}^4.$$

Matching this with I_{req} leads to:

$$2.4 \times 10^{-4} t_{\text{flange}} + 6.4 \times 10^{-9} \geq 1.2 \times 10^{-7} \Rightarrow t_{\text{flange}} \geq 0.473 \text{ mm}.$$

Let's choose the flange thickness to be 0.5 mm.

This makes the design second area moment of the section $I_{\text{des}} = 1.264 \times 10^{-7} \text{mm}^4$.

§2.3. Skin Reinforcements

The flange has been sized above based purely on material considerations. We left out compressive buckling intentionally. This is because designing the flange to avoid buckling will require much higher thickness, and we'd rather satisfy this by splitting the flange into sub panels for a more efficient design.

The design stress on the flange (at the root) comes out to be

$$\sigma_{\text{root flange}} = \frac{1000 \times 0.02}{1.264 \times 10^{-7}} = 158.23 \text{ MPa}.$$

The critical compressive buckling stress comes out to be

$$\sigma_{\text{crit}} = \frac{4}{3}\frac{\pi^2 E_y}{12(1-\nu^2)}\left(\frac{0.5}{300}\right)^2 = 234.42 \text{ kPa},$$

which is at least **3 orders of magnitude smaller than the actual stress**. Our workaround for this will be to add reinforcements on the flanges so that they are effectively "split" into smaller panels, i.e., with reduced base length. We will do this through the addition of equispaced "booms". By "boom" we just mean some added reinforcement that simulates a "locally pinned" behavior. This may be in the form of a small stiffener or even some corrugation in the skin itself. The general problem of sizing these booms will require a more detailed analysis of the buckling of reinforced plates and we will not go into that (see ch. 9 in [1] for a nice treatment). Our aim here is pedagogical so we shall take a simpler route.

Let's denote the "boom pitch" by b_{bp} and the section area of each boom by A_b. The total number of booms will be $n_b = 2 \times \frac{0.3}{b_{\text{bp}}} = \frac{0.6}{b_{\text{bp}}}$ (or some integer round-off of this quantity). Since all of these are 0.02 m away from the centroid, their cumulative contribution to the second area moment will be

$$I_{\text{booms}} = n_b \times 0.02^2 A_b = 2.4 \times 10^{-4}\frac{A_b}{b_{\text{bp}}}.$$

This revises the axial stress at the root flange, which gets written as:

$$\sigma_{\text{revised}} = \frac{2 \times 10^5}{1.264 \times 10^{-3} + 2.4\frac{A_b}{b_{\text{bp}}}}.$$

We shall require this stress to not exceed the buckling critical stress of the individual panels. This gets written mathematically as

$$\frac{2 \times 10^5}{1.264 \times 10^{-3} + 2.4\frac{A_b}{b_{\text{bp}}}} \leq \frac{4}{3}\frac{\pi^2 E_y}{12(1-\nu^2)}\left(\frac{0.0005}{b_{\text{bp}}}\right)^2.$$

It does't really make a lot of sense to have booms that are bigger than b_{bp}^2. So we introduce a constraint that $A_b < 0.5 b_{\text{bp}}^2$. With this we can start sizing the booms. Figure 5 depicts the sizing curves for the boom.

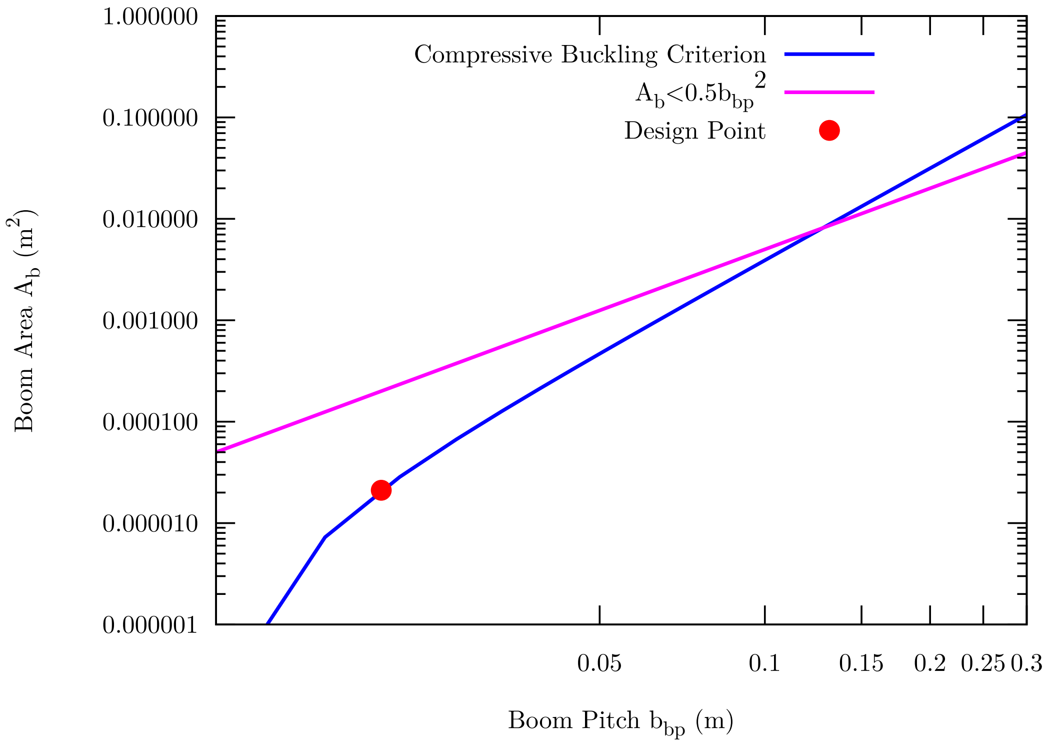

Figure 5: The Boom Sizing Curves - the first curve takes care of buckling, the second enforces our physical constraint. The design point corresponds to $b_{\text{bp}} = 20$ mm and $A_b = 21.1\text{mm}^2$.

Let us choose a boom pitch of 20 mm, so we will have 15 booms at the root on each flange.

As per this, the individual boom areas will be $A_b = 21.1\text{mm}^2$. For simplicity, let us assume that these are square sections with side 4.6 mm each. Now the total second area moment comes to $3.793 \times 10^{-7}\text{m}^4$ and the root flange stress is 52.7 MPa. **Note here that we have now sacrificed some weight just for the buckling reinforcement**. It will be a more optimal design if the material failure and buckling happen at the same stress level.

§2.4. Boom Reinforcement

Now the booms themselves can buckle, so we would like to reinforce them by adding intermediate ribs within the beam. For the square booms, the second area moments are $I_{\text{boom}} = 37.3\text{mm}^4$ so the critical buckling stress expression is

$$\sigma_{\text{crit}} = \frac{\pi^2 E_y I_{\text{boom}}}{3 b_{\text{rp}}^2 A_b} = \frac{4.08 \times 10^5}{b_{\text{rp}}^2},$$

where b_{rp} is the "rib pitch", i.e., the length between two ribs. Setting the above equal to the stress 52.7 MPa lets us choose $b_{\text{rp}} \leq 87.9$ mm.

We shall choose $b_{\text{rp}} = 83.33$ mm so that there is a total of 12 ribs along the span of the beam.

For simplicity, we can use the same rib as designed for the tip, but remember that this will be significantly heavier than it needs to be.

§3. Further Considerations

This brief white paper is only meant as an introduction to the design process. You can think along these lines and refine the design even further. Some avenues for optimizing include: reducing number of booms with span, making cutouts within the rib webs, etc. In any case, this is only meant as a pedagogical introduction to the design of thin-walled structures. [2] is excellent as a reference for design of aircraft structures.

Bibliography

[1] S. P. Timoshenko and J. M. Gere, *Theory of Elastic Stability*. Courier Corporation, 2009.

[2] E. F. Bruhn, *Analysis And Design Of Airplane Structures*. 1949.