The Michell Solution

AS2070, Dept. of Aerospace Engg., IIT Madras (Jan-May 2025)

ϕ	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$	$2\mu u_r$	$2\mu u_\theta$
r^2	2	0	2	$(\kappa-1)r$	0
$r^2\ln r$	$2\ln(r)+1$	0	$2\ln(r)+3$	$(\kappa-1)r\ln(r)-r$	$(\kappa+1)r\theta$
$\ln r$	$1/r^2$	0	$-1/r^2$	$-1/r$	0
θ	0	$1/r^2$	0	0	$-1/r$
$r\theta\sin\theta$	$2\cos\theta/r$	0	0	$\frac{1}{2}\left\{(\kappa-1)\theta\sin\theta - \cos\theta+(\kappa+1)\ln(r)\cos\theta\right\}$	$\frac{1}{2}\left\{(\kappa-1)\theta\cos\theta-\sin\theta -(\kappa+1)\ln(r)\sin\theta\right\}$
$r\ln(r)\cos\theta$	$\cos\theta/r$	$\sin\theta/r$	$\cos\theta/r$	$\frac{1}{2}\left\{(\kappa+1)\theta\sin\theta - \cos\theta+(\kappa-1)\ln(r)\cos\theta\right\}$	$\frac{1}{2}\left\{(\kappa+1)\theta\cos\theta-\sin\theta -(\kappa-1)\ln(r)\sin\theta\right\}$
$r\theta\cos\theta$	$-2\sin\theta/r$	0	0	$\frac{1}{2}\left\{(\kappa-1)\theta\cos\theta +\sin\theta-(\kappa+1)\ln(r)\sin\theta\right\}$	$\frac{1}{2}\left\{-(\kappa-1)\theta\sin\theta -\cos\theta-(\kappa+1)\ln(r)\cos\theta\right\}$
$r\ln(r)\sin\theta$	$\sin\theta/r$	$-\cos\theta/r$	$\sin\theta/r$	$\frac{1}{2}\left\{-(\kappa+1)\theta\cos\theta -\sin\theta+(\kappa-1)\ln(r)\sin\theta\right\}$	$\frac{1}{2}\left\{(\kappa+1)\theta\sin\theta +\cos\theta+(\kappa-1)\ln(r)\cos\theta\right\}$
$r^{n+2}\cos n\theta$	$-(n+1)(n-2)r^n\cos n\theta$	$n(n+1)r^n\sin n\theta$	$(n+1)(n+2)r^n\cos n\theta$	$(\kappa-n-1)r^{n+1}\cos n\theta$	$(\kappa+n+1)r^{n+1}\sin n\theta$
$r^n\cos n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$	$n(n-1)r^{n-2}\cos n\theta$	$-nr^{n-1}\cos n\theta$	$nr^{n-1}\sin n\theta$
$r^{n+2}\sin n\theta$	$-(n+1)(n-2)r^n\sin n\theta$	$-n(n+1)r^n\cos n\theta$	$(n+1)(n+2)r^n\sin n\theta$	$(\kappa-n-1)r^{n+1}\sin n\theta$	$-(\kappa+n+1)r^{n+1}\cos n\theta$
$r^n\sin n\theta$	$-n(n-1)r^{n-2}\sin n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$	$-nr^{n-1}\sin n\theta$	$-nr^{n-1}\cos n\theta$

with $\kappa = \begin{cases} 3-4\nu & \text{plane strain} \\ \frac{3-\nu}{1+\nu} & \text{plane stress} \end{cases}$. Rigid Body Motion: $\begin{bmatrix} u_r \\ u_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} 0 \\ r \end{bmatrix} C_3 = \begin{bmatrix} C_1\cos\theta + C_2\sin\theta \\ -C_1\sin\theta + C_2\cos\theta + rC_3 \end{bmatrix}$.

Transformation: $\begin{bmatrix} u_x \\ u_y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} u_r \\ u_\theta \end{bmatrix}$. Calculus: $\underline{\nabla} = \underline{e_r}\frac{\partial}{\partial r} + \underline{e_\theta}\frac{1}{r}\frac{\partial}{\partial\theta}$, $\quad \nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}$.