

AS3020: Aerospace Structures

Module 8: Some Results From Buckling (V3)

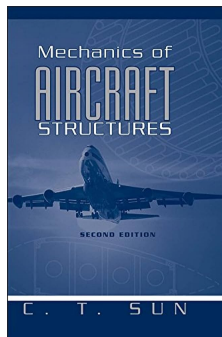
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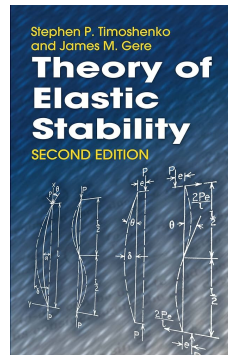
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Chapter 7 in Sun (2006)



Chapter 5 in Timoshenko and Gere (2009)

1.1. Transverse Buckling I

Prismatic Structures

- The transverse buckling equation you should be exposed to thus far starts with an assumed kinematic field consisting of pure bending (assuming Kirchhoff kinematic assumptions):

$$u_1 = -(X_2 v' + X_3 w'), \quad u_2 = v, \quad u_3 = w,$$

and an assumed “dominant” compressive stress field:

$$\underline{\underline{\sigma}}^P = \begin{bmatrix} \frac{-P}{A} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

with P being the applied load and A being the sectional area.

- The axial strain for the above deformation field is

$$\mathcal{E}_{11} = u_{1,1} + \frac{1}{2} \left(u_{1,1}^2 + u_{2,1}^2 + u_{3,1}^2 \right),$$

where we have used the full nonlinear expression for the Lagrangian strain.

1.1. Transverse Buckling II

Prismatic Structures

- We drop the $u_{1,1}^2$ term above for the small strain case (since $u_{1,1} \gg \frac{u_{1,1}^2}{2}$) and obtain the **Von Karman Strain Expression**:

$$\mathcal{E}_{11} = u_{1,1} + \frac{u_{2,1}^2 + u_{3,1}^2}{2}.$$

- The overall stress field will be taken as the sum of the imposed field of $\sigma_{P11} = -\frac{P}{A}$ and the elastic stress $\sigma_{11} = E_y \mathcal{E}_{11}$.
- The virtual work due to the stress (integrated over the section $\mathcal{S}$) is written as

$$\delta W = \int_{\mathcal{S}} \sigma_{11} \delta \mathcal{E}_{11} = \int_{\mathcal{S}} \left(-\frac{P}{A} + E_y \mathcal{E}_{11} \right) \delta \mathcal{E}_{11} = \int_{\mathcal{S}} \overbrace{-\frac{P}{A} \delta \mathcal{E}_{11}}^{-\delta \Pi} + \overbrace{E_y \mathcal{E}_{11} \delta \mathcal{E}_{11}}^{\delta U} = \delta(U - \Pi).$$

Where we have identified $\delta \Pi$ as the load contribution and δU as the elastic contribution.

1.1. Transverse Buckling III

Prismatic Structures

- The load contribution is written as

$$\delta\Pi = \frac{P}{A} \int_S \delta u_{1,1} + (u_{2,1}\delta u_{2,1} + u_{3,1}\delta u_{3,1})dA$$

which, upon substitution of the above displacement field and assuming centroidal coordinate system becomes,

$$\delta\Pi = P(v_{,1}\delta v_{,1} + w_{,1}\delta w_{,1}).$$

- We need to integrate this along the span of the beam to obtain the overall virtual work (let us assume $P(X_1)$). This simplifies through integration by parts as:

$$\begin{aligned} \int_0^\ell \delta\Pi &= \int_0^\ell P(v_{,1}\delta v_{,1} + w_{,1}\delta w_{,1})dX_1 \\ &= P(v_{,1}\delta v + w_{,1}\delta w)\Big|_0^\ell - \int_0^\ell ((Pv_{,1})_{,1}\delta v + (Pw_{,1})_{,1}\delta w) dX_1 \\ &= - \int_0^\ell \delta \underline{V}^T (P\underline{V}_{,1})_{,1} dX_1 + P\delta \underline{V}^T \underline{V}_{,1}\Big|_0^\ell \end{aligned}$$

1.1. Transverse Buckling IV

Prismatic Structures

- Next we consider the elastic contributions

$$\delta U = E_y \begin{bmatrix} \delta v_{,11} & \delta w_{,11} \end{bmatrix} \begin{bmatrix} I_{33} & I_{23} \\ I_{23} & I_{22} \end{bmatrix} \begin{bmatrix} v_{,11} \\ w_{,11} \end{bmatrix} = E_y \delta \underline{V}_{,11}^T \underline{I} \underline{V}_{,11}$$

which, upon span-wise integration, becomes

$$\int_0^\ell \delta U = \delta \underline{V}_{,1}^T E_y \underline{I} \underline{V}_{,11} \Big|_0^\ell - \delta \underline{V}^T \left(E_y \underline{I} \underline{V}_{,11} \right)_{,1} \Big|_0^\ell + \int_0^\ell \delta \underline{V}^T \left(E_y \underline{I} \underline{V}_{,11} \right)_{,11} dX_1.$$

- Putting everything together, we have

$$\begin{aligned} \int_0^\ell \delta W = & \int_0^\ell \delta \underline{V}^T \left(\left(E_y \underline{I} \underline{V}_{,11} \right)_{,11} + \left(P \underline{V}_{,1} \right)_{,1} \right) dX_1 \\ & - \delta \underline{V}^T \left(\left(E_y \underline{I} \underline{V}_{,11} \right)_{,1} + P \underline{V}_{,1} \right) \Big|_0^\ell + \delta \underline{V}'^T \left(E_y \underline{I} \underline{V}_{,11} \right) \Big|_0^\ell. \end{aligned}$$

By the principle of virtual work the above must be zero for equilibrium for arbitrary $\delta \underline{V}$ and $\delta \underline{V}_{,1}$.

1.1. Transverse Buckling V

Prismatic Structures

- For this equality to hold for arbitrary virtual displacements ($\delta \underline{V}$) and rotations ($\delta \underline{V}'$) each of the terms above should be equated to zero in their respective domains. So we obtain the differential equation system:

$$\begin{aligned} \left(E_y I_{\underline{\underline{V}}},_{11} \right)_{,11} + \left(P \underline{V},_1 \right)_{,1} &= 0, \quad X_1 \in (0, \ell) \\ \left(E_y I_{\underline{\underline{V}}},_{11} \right)_{,1} + P \underline{V},_1 &= 0, \quad (\text{OR}) \quad \underline{V} = \text{specified}, \quad X_1 \in \{0, \ell\} \\ E_y I_{\underline{\underline{V}}},_{11} &= 0, \quad (\text{OR}) \quad \underline{V}' = \text{specified}, \quad X_1 \in \{0, \ell\}. \end{aligned}$$

Note that this assumes that no other form of external load is applied.

- Considering the $\underline{e}_2$ deformation in the symmetric case ($I_{23} = 0$), the above simplifies to

$$\boxed{E_y I_{33} v'''' + P v'' = 0},$$

which is the familiar Euler equation for buckling.

- Refer to ch. 7 in Sun (2006) for the different cases of boundary conditions herein.

1.2. Torsional Buckling I

Prismatic Structures

- For the transverse case, we started with a pure bending kinematics and derived the transverse load due to compression using the work done. **What happens when we also account for torsion?**
- Here, the kinematic deformation field is

$$u_1 = \theta_{,1}\psi, \quad u_2 = -X_3\theta, \quad u_3 = X_2\theta,$$

where $\theta(X_1)$ is the twisting angle (we allow this to be a general function of X_1), and $\psi(X_2, X_3)$ is the St-Venant warping function (see Module 5).

- The axial strain (under Von Karman simplification, as before) is:

$$\mathcal{E}_{11} = u_{1,1} + \frac{1}{2}(u_{2,1}^2 + u_{3,1}^2) = \psi\theta_{,11} + \frac{X_2^2 + X_3^2}{2}\theta_{,1}^2.$$

- The shear strains (we only write out the linear strains) are

$$\gamma_{12} = (\psi_{,2} - X_3)\theta_{,1}, \quad \eta_{13} = (\psi_{,3} + X_2)\theta_{,1}.$$

1.2. Torsional Buckling II

Prismatic Structures

- Following the same process as above, the virtual work under an axial imposed stress field of $\sigma_{11} = -\frac{P}{A}$ is written as

$$\begin{aligned}\delta W &= \int_S \left(-\frac{P}{A} + E_y \mathcal{E}_{11}\right) \delta \mathcal{E}_{11} + G \gamma_{12} \delta \gamma_{12} + G \gamma_{13} \delta \gamma_{13} \\ &= \int_S \underbrace{-\frac{P}{A} \delta \mathcal{E}_{11}}_{-\delta \Pi} + \underbrace{E_y \mathcal{E}_{11} \delta \mathcal{E}_{11} + G \gamma_{12} \delta \gamma_{12} + G \gamma_{13} \delta \gamma_{13}}_{\delta U} = \delta(U - \Pi).\end{aligned}$$

- Note that unlike the bending case, we have considered shear contributions also here.
- The load contribution is written as

$$\delta \Pi = \frac{P}{A} \int_S \psi \delta \theta_{,11} + (X_2^2 + X_3^2) \theta_{,1} \delta \theta_{,1} dA = \frac{P}{A} \delta \theta_{,11} \int_S \cancel{\psi dA} + P \frac{I_{11}}{A} \theta_{,1} \delta \theta_{,1},$$

where we have canceled out $\int_S \psi dA$ because net displacement due to warping is zero. I_{11} is the polar second moment of area.

- We next consider the integral of $\delta \Pi$ and integrate it by parts to write:

$$\int_0^\ell \delta \Pi = \int_0^\ell \frac{P I_{11}}{A} \theta_{,1} \delta \theta_{,1} dX_1 = \frac{P I_{11}}{A} \theta_{,1} \delta \theta \Big|_0^\ell - \int_0^\ell \left(\frac{P I_{11}}{A} \theta_{,1} \right)_{,1} dX_1.$$

1.2. Torsional Buckling III

Prismatic Structures

- Coming to the elastic contributions, this simplifies as

$$\begin{aligned}
 \delta U &= \int_S E_y \psi^2 \theta_{,11} \delta \theta_{,11} + G \left((\psi_{,2} - X_3)^2 + (\psi_{,3} + X_2)^2 \right) \theta_{,1} \delta \theta_{,1} \\
 &= \int_S E_y \psi^2 dA \theta_{,11} \delta \theta_{,11} + \int_S G \left(\underbrace{\psi_{,2}^2 + \psi_{,3}^2}_{\substack{0 \\ -\int_S \psi \nabla^2 \psi + \int_{\partial S} \psi \nabla \psi \cdot \underline{n}}} + X_2^2 + X_3^2 + 2X_2 \psi_{,3} - 2X_3 \psi_{,2} \right) dA \theta_{,1} \delta \theta_{,1} \\
 &= \underbrace{\int_S E_y \psi^2 dA}_{:= E_y C_w} \theta_{,11} \delta \theta_{,11} + \underbrace{G \left(I_{11} + \int_S X_2 \psi_{,3} - X_3 \psi_{,2} dA \right)}_{:= GJ} \theta_{,1} \delta \theta_{,1} \\
 &= E_y C_w \theta_{,11} \delta \theta_{,11} + GJ \theta_{,1} \delta \theta_{,1}.
 \end{aligned}$$

- Here, C_w is defined as the **warping constant** of the section, and is a property of the section (like area, second moment, etc.).

1.2. Torsional Buckling IV

Prismatic Structures

- Integrating δU over the span of the beam leads to

$$\begin{aligned} \int_0^\ell \delta U = & E_y C_w \theta_{,11} \delta \theta_{,1} \Big|_0^\ell - \left((E_y C_w \theta_{,11})_{,1} - GJ \theta_{,1} \right) \delta \theta \Big|_0^\ell \\ & + \int_0^\ell \left((E_y C_w \theta_{,11})_{,11} - (GJ \theta_{,1})_{,1} \right) \delta \theta dX_1 \end{aligned}$$

- Now we put everything together.

$$\begin{aligned} \int_0^\ell \delta W = & \int_0^\ell \left((E_y C_w \theta_{,11})_{,11} - (GJ \theta_{,1})_{,1} + \left(\frac{P I_{11}}{A} \theta_{,1} \right)_{,1} \right) \delta \theta dX_1 \\ & + \left[E_y C_w \theta_{,11} \delta \theta_{,1} - \left((E_y C_w \theta_{,11})_{,1} - GJ \theta_{,1} + \frac{P I_{11}}{A} \theta_{,1} \right) \delta \theta \right] \Big|_0^\ell \end{aligned}$$

1.2. Torsional Buckling V

Prismatic Structures

- The results in the following PDE governing torsional deformation under axial load:

$$(E_y C_w \theta_{,11})_{,11} + \left(\left(\frac{P I_{11}}{A} - GJ \right) \theta_{,1} \right)_{,1} = 0, \quad X_1 \in (0, \ell)$$

$$E_y C_w \theta_{,11} = 0, \quad (\text{OR}) \quad \theta_{,1} = \text{specified}, \quad X_1 \in \{0, \ell\}$$

$$(E_y C_w \theta_{,11})_{,1} + \left(\frac{P I_{11}}{A} - GJ \right) \theta_{,1} = 0, \quad (\text{OR}) \quad \theta = \text{specified}, \quad X_1 \in \{0, \ell\}.$$

- For constant parameters (uniform prismatic beam) the governing equation is written as:

$$E_y C_w \theta_{,1111} + \left(\frac{P I_{11}}{A} - GJ \right) \theta_{,11} = 0,$$

which is a Sturm-Liouville equation of the form

$$\theta_{,1111} + p^2 \theta_{,11} = 0, \quad p^2 = \frac{\frac{P I_{11}}{A} - GJ}{E_y C_w}$$

which is mathematically identical to the transverse buckling equation.

- Even boundary conditions are very similar, so the solution procedure will follow exactly the same process.

1.2. Torsional Buckling: Alternative Derivation through Shear Flow Arguments I

Prismatic Structures

- It is helpful to use a more “practical” derivation to understand the $E_y C_w$ term better. We shall revert to shear flow for this - it turns out that the $E_y C_w$ term is due to the shear flow variations “induced” by the axial stress field that the warping function induces.
- From the consideration of “pure twist”, the twisting moment is written as

$$M_{twist} = GJ\theta_{,1}$$

as per the notation introduced in Module 5. Recall that we had $J = I_{11} - \frac{1}{2} \int_{\partial S} \frac{\partial \psi^2}{\partial n} d\ell$ for the torsion constant.

- We ignored the straight stresses σ_{11} in Module 5 since $\int_S \sigma_{11} dA = 0$ and we argued that σ_{11} will be small. Although small, the variations are sufficient to induce additional shear flow, so let us consider it now.
- Using linear elasticity we have

$$\sigma_{11} = E_y \mathcal{E}_{11} = E_y \theta_{,11} \psi.$$

1.2. Torsional Buckling: Alternative Derivation through Shear Flow Arguments II

Prismatic Structures

- The shear flow that the section develops is written as (same procedure as we followed in Modules 4 and 5):

$$\frac{dq}{ds} + t\sigma_{11,1} = 0 \implies q_{warp}(s) - q_0 = - \int_0^s t\sigma_{11,1} ds = -E_y\theta_{,111} \int_0^s t\psi ds.$$

- Let us restrict our discussions to open sections with $q_0 = 0$ (our running integral starts from a free tip). Recall that our analysis in Module 5 led to:
 $\psi(s) = -2A_{Os}(s) = - \int_0^s p(s) ds.$
- The twisting moment that the warping shear flow $q_{warp}(s)$ leads to is written as the full integral (denoted $\int (\cdot) ds$)

$$M_{warp} = \int p(s)q_{warp}(s)ds = \int \left(-\frac{d\psi}{ds} \right) \left(\int_0^s -E_y\theta_{,111}t\psi(z)dz \right) ds,$$

where we have invoked $p(s) = -\frac{d\psi}{ds}$.

1.2. Torsional Buckling: Alternative Derivation through Shear Flow Arguments III

Prismatic Structures

- Applying integration by parts we have

$$M_{warp} = E_y \theta_{,11} \int \frac{d}{ds} \left(\psi(s) \int_0^s t \psi(z) dz \right) - t \psi^2(s) ds = -E_y \underbrace{\left(\int t \psi^2(s) ds \right)}_{C_w} \theta_{,11}$$

$$\Rightarrow \boxed{M_{warp} = -E_y C_w \theta_{,11}}.$$

The constant C_w is called the **warping constant** and is a sectional property (much like the warping function itself, torsion constant J , area A , second moments, etc.).

- Now we have a contribution from pure twist, $GJ\theta_{,1}$, and a contribution from warping suppression, $-E_y C_w \theta_{,11}$. Adding these both should give us the externally applied total moment:

$$\boxed{M_{tot} = GJ\theta_{,1} - E_y C_w \theta_{,11}}.$$

- For constant M_{tot} , the above can be solved to obtain $\theta(X_1)$. **This is the generalized equation governing torsional kinematics.**

1.2. Torsional Buckling: Alternative Derivation through Shear Flow Arguments IV

Prismatic Structures

- Under the presence of axial compression, $\frac{dM_{tot}}{dX_1} = m_P = -\frac{PI_{11}}{A}\theta_{,11}$. Incorporating this into the equation yields:

$$E_y C_w \theta_{,1111} + \left(P \frac{I_{11}}{A} - GJ \right) \theta_{,11} = 0,$$

which is identical to the equation we derived earlier.

- This version of the derivation is only to provide you an intuitive understanding of the extra term through shear flow. This does not mean that the $E_y C_w$ term is restricted to the thin walled open section case. Since the term follows from the more formal virtual work principle, this is quite general and we can compute the $E_y C_w$ term for any arbitrary solid section/closed section too:**

$$E_y C_w = \int_S E_y \psi^2 dA.$$

1.2. Torsional Buckling

Prismatic Structures

- The torsion constant J and warping constant C_w for common thin-walled sections are shown in the table here.

	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2 b^3}{24}$
	$e = h - \frac{b_1^3}{b_1^3 + b_2^3}$ $J = \frac{(b_1 + b_2)t_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f h^2}{12} \frac{b_1^3 b_2^3}{b_1^3 + b_2^3}$
	$e = \frac{3b^2 t_f}{6bt_f + ht_w}$ $J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{t_f b^3 h^2}{12} \frac{3bt_f + 2ht_w}{6bt_f + ht_w}$
	$J = \frac{2bt_f^3 + ht_w^3}{3}$ $C_w = \frac{b^3 h^2}{12(2b + h)^2} \times [2t_f(b^2 + bh + h^2) + 3t_w bh]$
	$e = 2a \frac{\sin \alpha - \alpha \cos \alpha}{\alpha - \sin \alpha \cos \alpha}$ $J = \frac{2aat^3}{3}$ $C_w = \frac{2ta^3}{3} \times \left[\alpha^3 - \frac{6(\sin \alpha - \alpha \cos \alpha)^2}{\alpha - \sin \alpha \cos \alpha} \right]$

^aO is shear center.

Table 7.2 from Sun (2006)

1.2.1. Torsional Buckling: The Simply Supported Case I

Prismatic Structures

- The simplest example is the simply supported beam under axial compression. Transverse buckling leads to:

$$P_{cr-tr,n} = n^2 \frac{\pi^2 E_y I_p}{\ell^2}$$

where I_p is a principal second moment (i.e., eigenvalue of the $\underline{I}$ matrix).

- We have already seen how the torsion buckling problem is mathematically identical to the flexural buckling problem, so the critical load can directly be written as,

$$P_{cr-tw,n} \frac{I_{11}}{A} - GJ = n^2 \frac{\pi^2 E_y C_w}{\ell^2} \implies P_{cr-tw,n} = GJ \frac{A}{I_{11}} + n^2 \frac{A}{I_{11}} \frac{\pi^2 E_y C_w}{\ell^2}.$$

- We now have **two different critical load estimates!**
 - When P exceeds $P_{cr-tr,n}$, then the beam **buckles transversely**, or bends.
 - When P exceeds $P_{cr-tw,n}$, then the beam **twists**, or undergoes torsion deformation.
- A real designer must account for both. Let us set $n = 1$ and find when the twist-buckling will occur at a lower load than transverse buckling:

$$P_{cr-tw,1} \leq P_{cr-tr,1} \implies GJ \leq \frac{\pi^2 E_y}{\ell^2} \left(\frac{I_{11}}{A} I_p - C_w \right).$$

1.2.1. Torsional Buckling: The Simply Supported Case II

Prismatic Structures

- The RHS in the above represents a limiting value for the torsional rigidity of open sections. As it is, recall that $J \sim \mathcal{O}(t^3)$ for open thin walled sections, so it is not very difficult to meet the above condition in the open section case. (It is a little more difficult for closed sections)
- In practice, we find that the warping constant C_w is small/close to zero for **thin walled rectangular sections meeting at a single point** like the following.

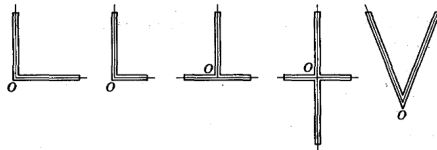


Fig. 5-5 from Timoshenko and Gere (2009)

1.2.1. Torsional Buckling: The Simply Supported Case III

Prismatic Structures

- So the condition can be simplified to

$$GJ \leq \frac{I_{11}}{A} P_{cr-tr,1}$$

where $P_{cr-tr,1} = \frac{\pi^2 E_y I_p}{\ell^2}$ is the Euler critical load.

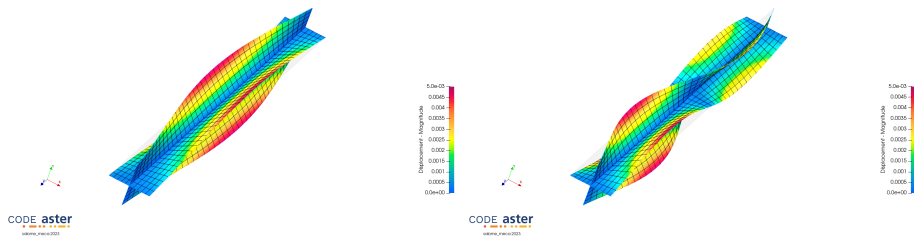
- This form of the equation is valid even for other boundary conditions - you just substitute the correct critical load.
- Furthermore, substituting $C_w = 0$ in the critical load expression leads to

$$P_{cr-tw,n} = GJ \frac{A}{I_{11}}, \text{ i.e., the (first) critical load is independent of the length of the beam!}$$

(Note that the $C_w = 0$ approximation no longer holds for the higher modes)

1.2.2. Torsional Buckling: Deformation Fields for a "Cross" Section

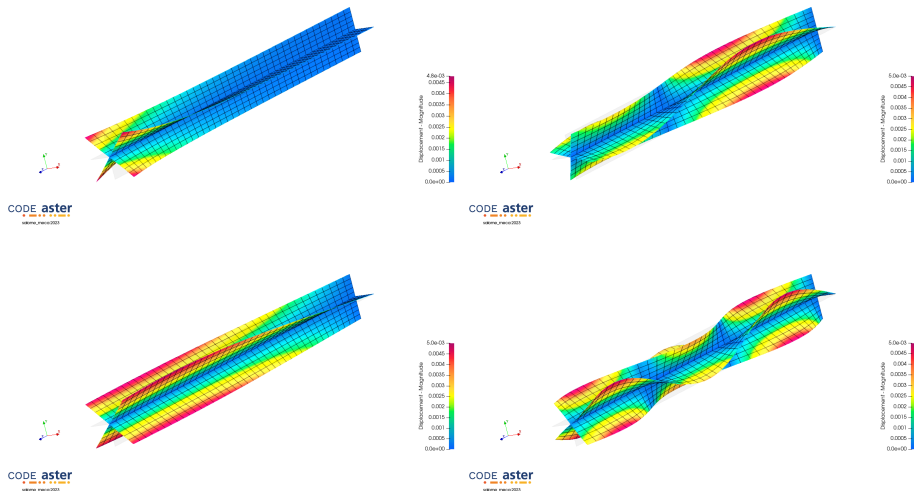
Prismatic Structures



First Two Buckling Modes in the Simply Supported Condition

1.2.2. Torsional Buckling: Deformation Fields for a "Cross" Section

Prismatic Structures



First Four Buckling Modes in the Cantilevered Condition

2.1. Buckling of Plates I

Planar Structures

- Let us consider a plate on the $\underline{e}_1 - \underline{e}_2$ plane. Invoking Kirchhoff assumptions (zero shear strain, just like we did with beams), the displacement field can be written as

$$u_1 = -X_3 w_{,1}, \quad u_2 = -X_3 w_{,2}, \quad u_3 = w.$$

- The relevant (Von Karman) strains are,

$$\begin{aligned} \mathcal{E}_{11} &= u_{1,1} + \frac{u_{3,1}^2}{2} &= -X_3 w_{,11} + \frac{w_{,1}^2}{2} \\ \mathcal{E}_{22} &= u_{2,2} + \frac{u_{3,2}^2}{2} &= -X_3 w_{,22} + \frac{w_{,2}^2}{2} \\ \gamma_{12} &= u_{1,2} + u_{2,1} + u_{3,1}u_{3,2} &= -X_3 2w_{,12} + w_{,1}w_{,2}. \end{aligned}$$

- The overall stress field is written as

$$\sigma_{11} = \frac{N_{11}}{t} + \frac{1}{E}(\mathcal{E}_{11} - \nu \mathcal{E}_{22}), \quad \sigma_{22} = \frac{N_{22}}{t} + \frac{1}{E}(\mathcal{E}_{22} - \nu \mathcal{E}_{11}), \quad \sigma_{12} = \frac{N_{12}}{t} + \frac{1}{G}\gamma_{12},$$

where N_1, N_2, N_{12} are all loads per unit length.

2.1. Buckling of Plates II

Planar Structures

- Skipping a few steps, we have the load contribution to the work coming purely from the combination of the imposed

$$\delta\Pi = N_{ij}w_{,i}\delta w_{,j}.$$

- Integrating this over the whole plate we have,

$$\int_{\mathcal{P}} \delta\Pi = \int_{\mathcal{P}} (N_{ij}w_{,i}\delta w)_{,j} - (N_{ij}w_{,i})_{,j}\delta w dA = - \int_{\mathcal{P}} (N_{ij}w_{,i})_{,j}\delta w dA + \int_{\partial\mathcal{P}} N_{ij}w_{,i}\delta w d\ell.$$

- The elastic contributions read:

References I

- [1] C. T. Sun. [Mechanics of Aircraft Structures](#), 2nd edition. Hoboken, N.J: Wiley, June 2006. ISBN: 978-0-471-69966-8 (cit. on pp. [2](#), [7](#), [17](#)).
- [2] Stephen P. Timoshenko and James M. Gere. [Theory of Elastic Stability](#), Courier Corporation, June 2009. ISBN: 978-0-486-47207-2 (cit. on pp. [2](#), [19](#)).