IMAC 2025, Orlando, FL, USA

#18782: Epistemic Insights on Frictional Hysteresis Models based on Micro-Slip Measurements under Single- and Multi-Harmonic Loading

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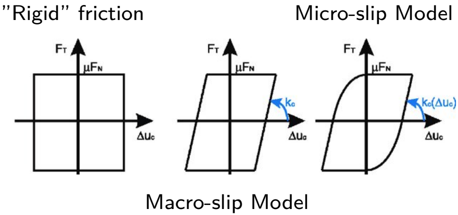
1. Introduction

- Frictional Joints are ubiquitous
- Two broad categories:
  Macro-Slip
  Micro-Slip ✓
- Different mathematical models have been proposed for each[a].
- Utility of these models in epistemic investigations of friction currently limited.

[a] Mathis et al. 2020.



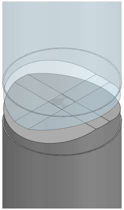
(Figure source: Moneta 2022)

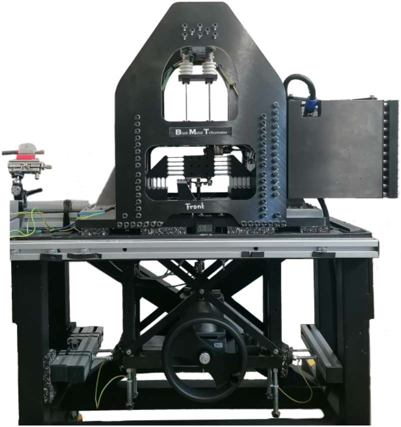
Goals

- Investigate *epistemic* features of experimentally measured hysteretic responses **in the micro-slip regime**
- Explore mathematical descriptions and **repeatable parametric identification** methodologies.

2. Experimental Campaign

- Tribometer designed to investigated friction in the **micro-slip regimes**
- Metallic (steel) specimens of $1 - 2$ mm^2 nominal contact area.





See talk #18786 for details

2. Experimental Campaign

- Voltage excitation of the form

$$\boxed{v(t) = V_1 \cos \Omega t + V_2 \cos(n\Omega t + \phi)}$$

- We have **five control parameters**:

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V_1, V_2 Harmonic amplitude (s)

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n, ϕ Higher harmonic and relative phase

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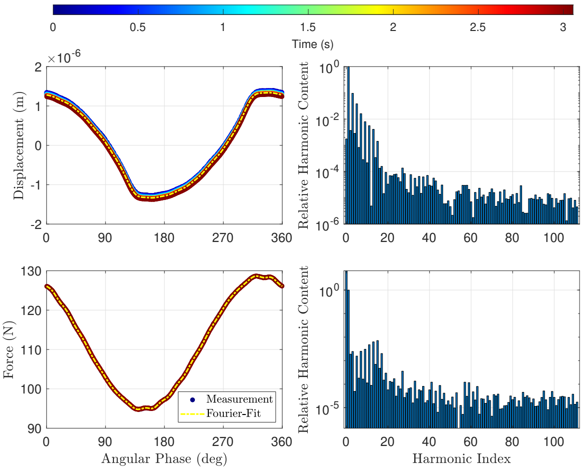
  Ω Excitation frequency
  V_1, V_2 Harmonic amplitude (s)
  n, ϕ Higher harmonic and relative phase

Test	V_1	V_2	Ω	n	ϕ	Remarks
1	1	0	20	3	0	Periodic excitation
2	0	1	20	3	0	
3	0.6	0.35	20	3	0	Varying second component amplitude and phase.
4	0.6	0.5	20	3	0	
5	0.6	0.6	20	3	90	
6	0.6	0.7	20	3	180	
7	0.6	0.7	20	3	232	
8	0.6	0.85	20	3	0	
9	0.6	0.85	20	3	135	
10	0.6	0.85	20	3	270	
11	0.6	0.85	20	2	0	Varying harmonic of second component.
12	0.6	0.85	20	4	315	
13	0.6	0.85	20	5	315	
14	0.6	0.85	20	6	0	
15	1	0	30	3	0	Periodic excitation cases with varying frequency.
16	1	0	40	3	0	
17	1	0	50	3	0	
18	1	0	60	3	0	
19	1	0	70	3	0	
20	1	0	80	3	0	

2. Experimental Campaign

Measurements are **Fourier-filtered**
with **111 harmonics** before analysis.



- Voltag

$v(t) =$

- We ha

Ω Exc

V_1, V_2

n, ϕ H
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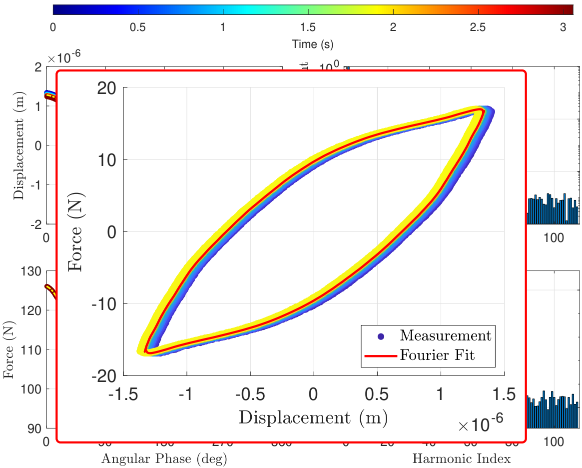

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3. Measurement Insights

Focus Aspects

Symmetry Masing's Hypotheses

Rate dependence, Normal Load Dependence Friction Modeling

Slip sensitivity Wear Degradation

Multi-Harmonic Responses Response Regimes

3.1. Rate Dependence

Measurement Insights

- We investigate rate dependence by **fixing voltage amplitude** and **varying frequency**
- This can help in the choice of an appropriate mathematical description

Rate Independence

Force-displacement relationship is **independent of the magnitude of velocity**.

3.1. Rate Dependence

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- We investigate rate dependence by **fixing voltage amplitude** and **varying frequency**
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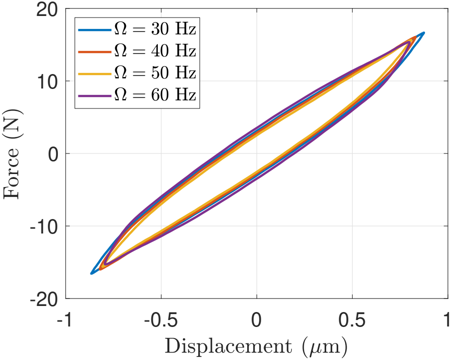
Rate Independence

Force-displacement relationship is **independent of the magnitude of velocity**.

While the responses are rate-independent for the most part, **relatively minor rate-dependence is observable**.



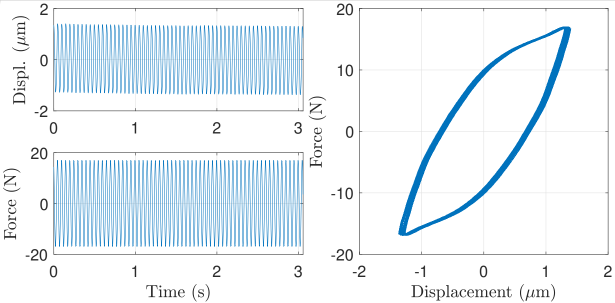
3.2. Symmetry

Measurement Insights

The Masing Element

- The **forward part** of the hysteresis curve is **identical to the reverse part** of the hysteresis curve, only **stretched** by a factor of two and **reflected** across the axes when oscillating between two extremes.
- The equation of any hysteretic response curve is **determined from the last point of the loading cycle before reversal** and requiring that if the loading curve crosses a previous loading curve, it must correspond to the previous loading curve.

As in (Brake 2017)



Raw Measurements, 20 Hz excitation

3.2. Symmetry

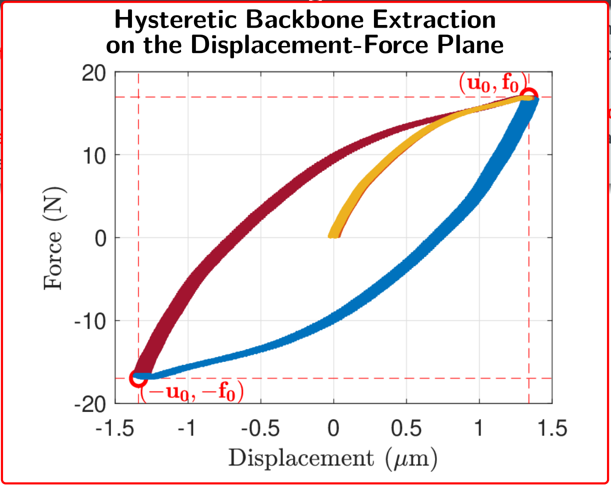
Measurement Insights

The Masing Element

- The **forward** … curve, only **s**… between two …
- The equation… **loading cycle**… loading curve…

…e hysteresis … oscillating … **oint of the** … previous …

As in (Brake 2017)



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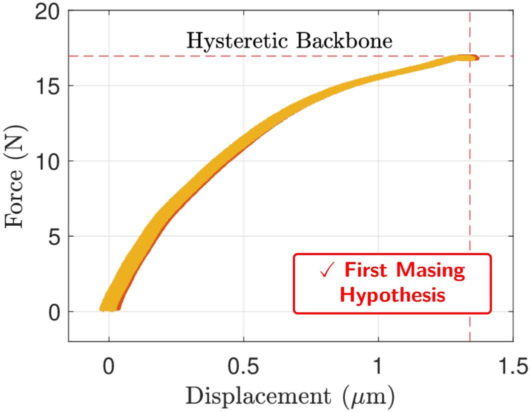
As in (Brake 2017)

Hysteretic Backbone Extraction on the Displacement-Force Plane



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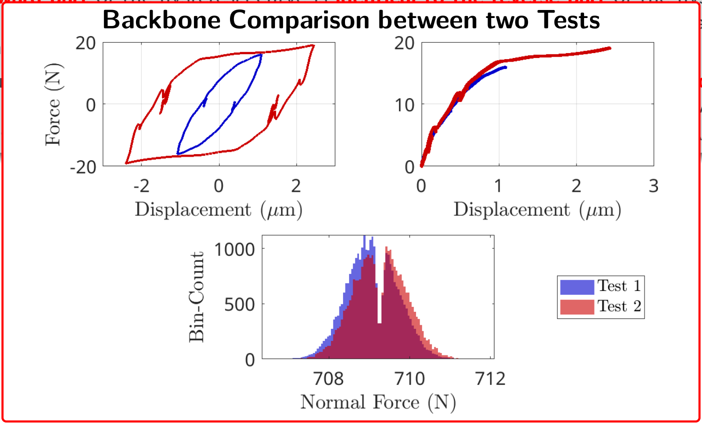

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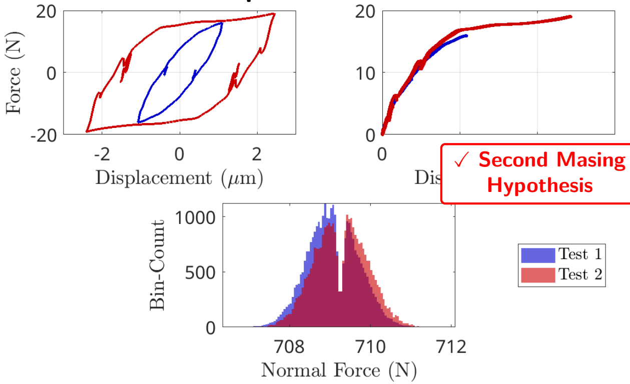
… s in (Brake 2017)

Backbone Comparison between two Tests

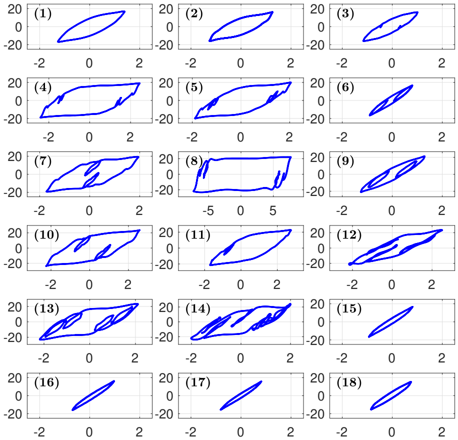


✓ Second Masing Hypothesis

3.3. Multi-Harmonic Responses

Measurement Insights

- Different parameter sets explore single and multi-harmonic response cases
- Hysteretic sub-cycles observed in the micro-slip regime

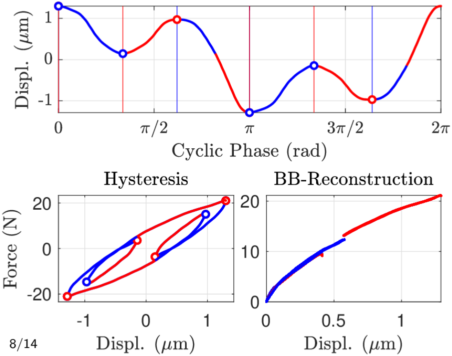


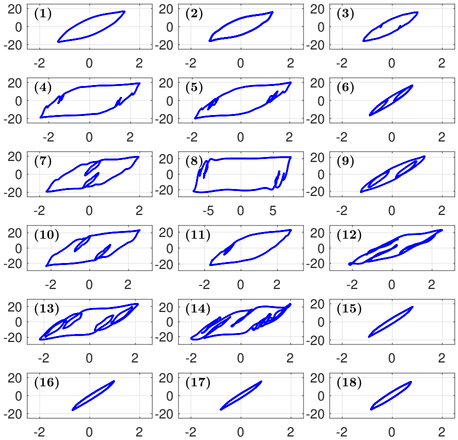
Force (N) vs Displacement (μm) Measurements

3.3. Multi-Harmonic Responses

Measurement Insights

- Different parameter sets explore single and multi-harmonic response cases
- Hysteretic sub-cycles observed in the micro-slip regime
- **Backbone is reconstructed** through careful post-processing





Force (N) vs Displacement (μm) Measurements

3.3. Multi-l

Measurement Insig

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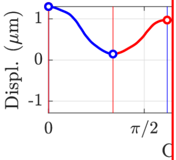
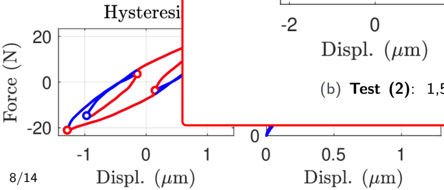
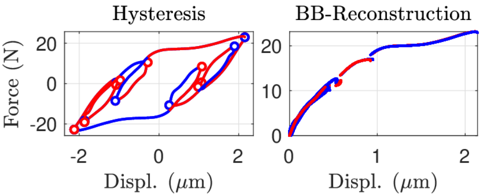
- Hysteretic sub-cy micro-slip regime
- **Backbone is rec** post-processing

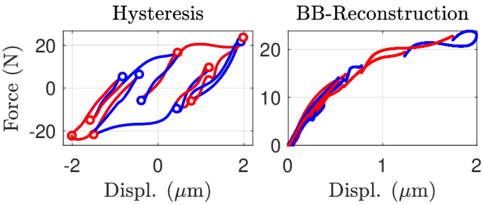




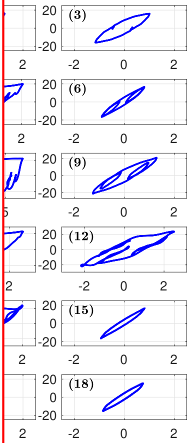
Higher harmonic examples



(a) **Test (1)**: 1,4 harmonics co-excited



(b) **Test (2)**: 1,5 harmonics co-excited

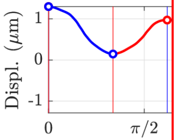


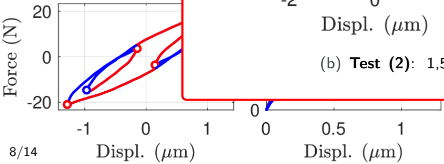
µm) Measurements

3.3. Multi-I

Measurement Insig

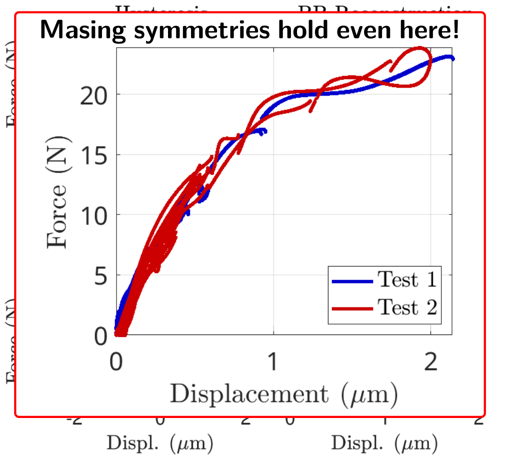
- Different parame multi-harmonic
- Hysteretic sub-c micro-slip regim
- **Backbone is rec** post-processing





Higher harmonic examples

Masing symmetries hold even here!



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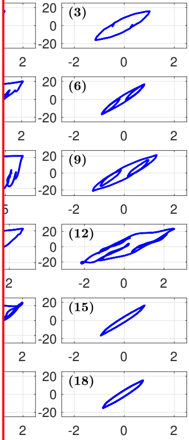
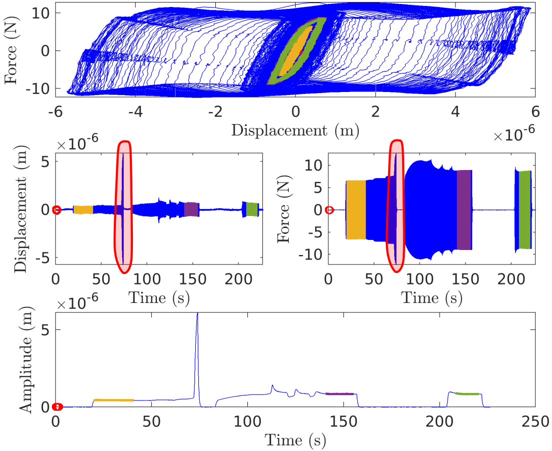


μm) Measurements

3.4. Slip Sensitivity

Measurement Insights

- The measured hystereses **extremely sensitive** to the occurrence of a **macro-slip event**



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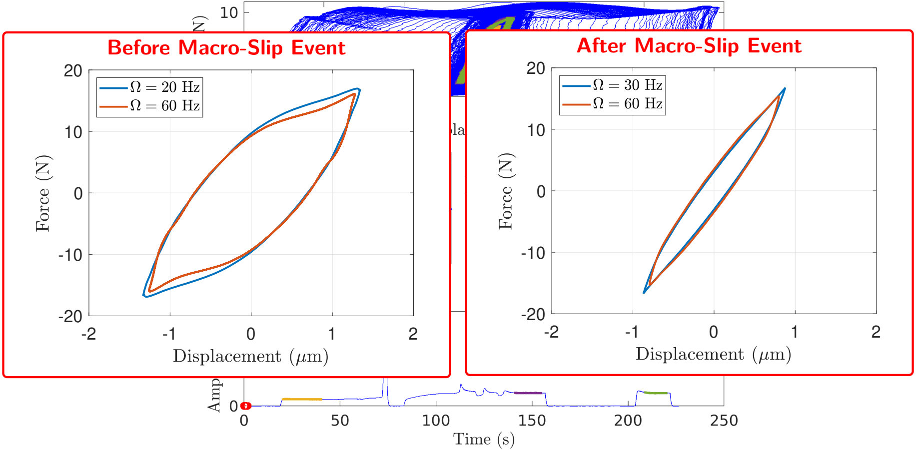
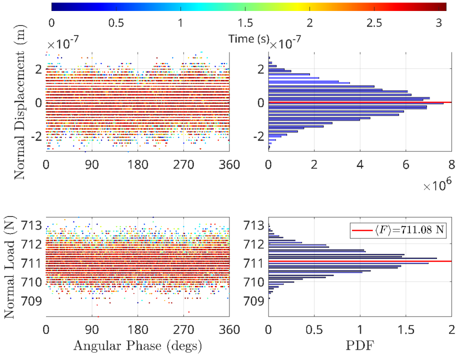


3.5. Normal Load Dependence

Measurement Insights

- There is currently no active control of normal load in our setup, **but this is measured**.
- We use this to study/observe **normal load dependence**.

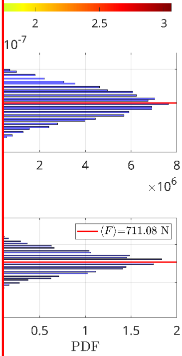


Measured normal displacement and load

3.5. Normal Load Dependence

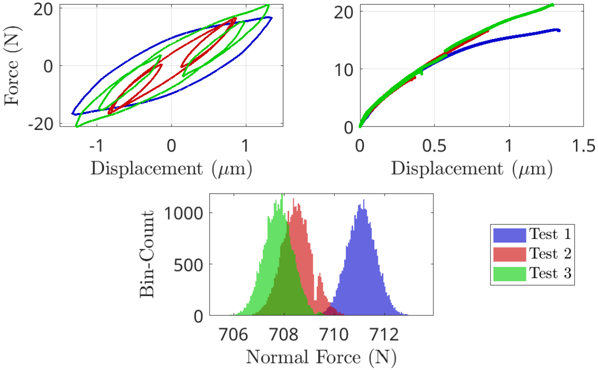
Measurement Insights

- There is cur
  load in our s
- We use this
  dependence



ment and load

Reconstructed Backbone Comparisons

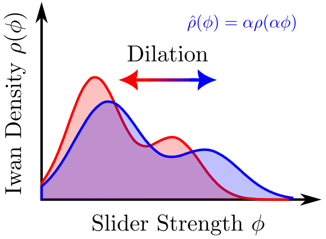


- Low amplitude regimes identical, divergences only later.

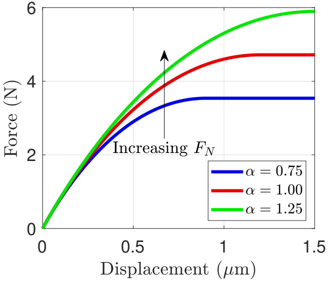
3.5. Normal Load Dependence

Measurement Insights

- There is currently no active control of normal load in our setup, **but this is measured**.
- We use this to study/observe **normal load dependence**.
- Theoretical studies in the Iwan framework (Rajaei and Ahmadian 2014) indicate that:
  - Linearized stiffness has no variation.
  - Macro-slip level scales.
  - **Scaling proportional to normal load.**



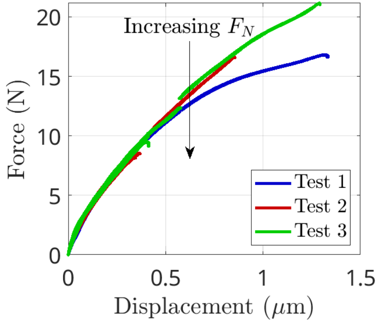
The 4-parameter Iwan under varying normal load (Rajaei and Ahmadian 2014).



3.5. Normal Load Dependence

Measurement Insights

Experimentally Reconstructed Backbones



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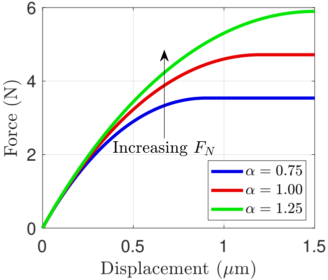


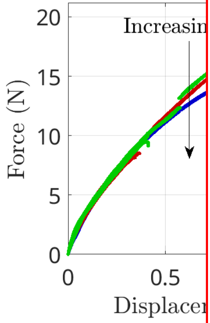
Seemingly Counter-Intuitive?

$$\overbrace{\uparrow F_N, \quad \downarrow \mu F_N}^{\text{Experimental Observation}} \quad \boxed{\Longrightarrow \downarrow \mu} \, ?$$

3.5. Normal Lo

Measurement Insights

Experime
Reconstructed



The coefficient of friction has a complex relationship to normal pressure

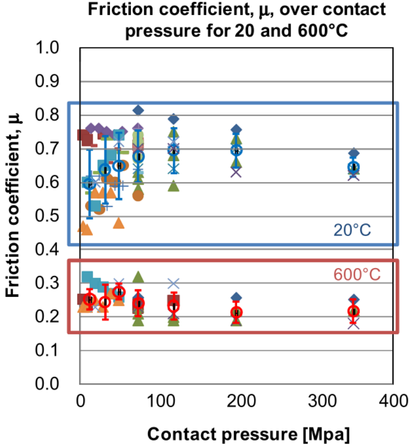
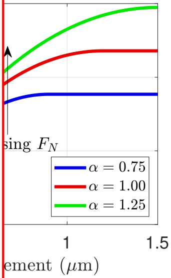


Figure from (Schwingshackl, Petrov, and Ewins 2012)

eter Iwan un-
normal load
hmadian 2014).



$$\overbrace{\uparrow F_N, \quad \downarrow \mu F_N}^{\text{Experimental Observation}} \quad \boxed{\implies \downarrow \mu} \; ?$$

4. Parametric Identification

Dynamic Regression: The Dahl Model

- **Time series of force is fit to the Dahl model:**

$$\dot{f} = \sigma_0 \left| 1 - \frac{f}{f_C}\mathrm{sgn}(\dot{u}) \right|^n \mathrm{sgn}\left(1 - \frac{f}{f_C}\mathrm{sgn}(\dot{u})\right) \dot{u}$$

- Linear Regression Framework ($n = 1$):

$$\dot{f} = \begin{bmatrix} \dot{u} & -f|\dot{u}| \end{bmatrix} \begin{bmatrix} \sigma_0 \\ f_C^{-1} \end{bmatrix}$$

- Parameters refined through nonlinear regression using these estimates.

Backbone Regression: 4-Parameter Iwan

- **Reconstructed backbones are fit to the 4-parameter Iwan model:**

$$F(u) = K_t u - \frac{R}{(\chi+1)(\chi+2)} u^{\chi+2}$$

- **Data assumed to be in micro-slip**.
- Integral Regression Formulation

$$I_F(u) = \begin{bmatrix} F(u)u & u^2 \end{bmatrix} \begin{bmatrix} \frac{1}{\chi+3} \\ \frac{Kt}{2}\frac{\chi+1}{\chi+3} \end{bmatrix}$$

List of Parameters

σ_0 Linearized stiffness

f_C Saturation force

n constant

List of Parameters

K_t Linearized stiffness

F_s Saturation load ($\beta = 0$ here.)

χ Dissipation power law exponent

4. Paran

Dynamic Regr … Iwan

- **Time ser … re fit to the model:**

$\dot{f} = \sigma_0 \Big| 1$ … $u^{\chi+2}$ … $+2)$

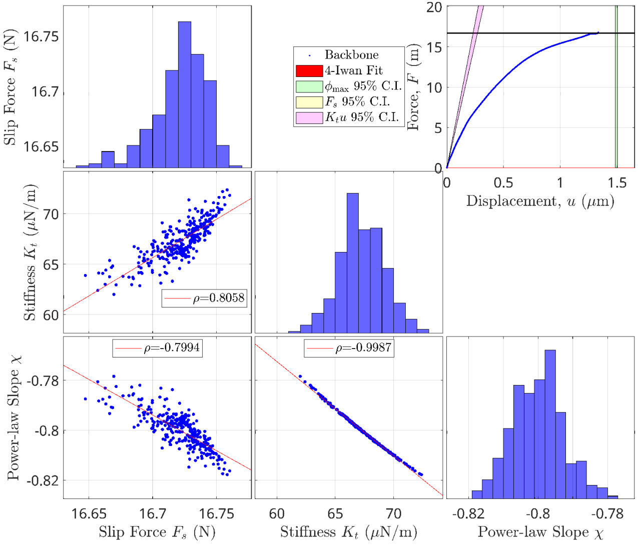
- Linear Re … **ro-slip**. … tion

$\frac{1}{\chi+3}$ … $\frac{K_t}{2} \frac{\chi+1}{\chi+3}$

- Paramete … regression

σ_0 Lineariz…
f_c Saturatio… ($\beta = 0$ here.)
n constant … ent

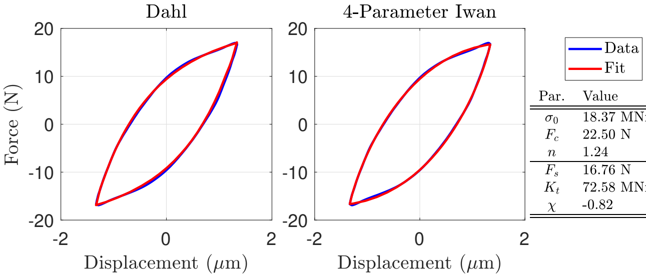
Advantage with Linear Regression:
Deterministic Repeatability in the Parameters



4.1. Model Form Assessment

Parametric Identification

- No differences observed when fitting single harmonic data



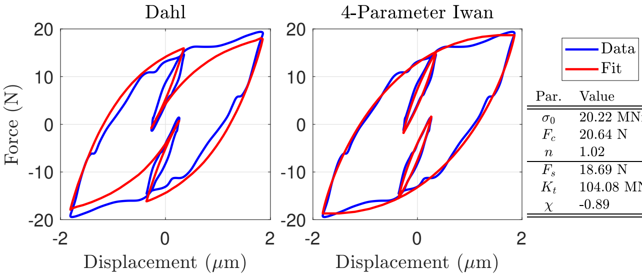
Par.	Value
σ_0	18.37 MNm^{-1}
F_c	22.50 N
n	1.24
F_s	16.76 N
K_t	72.58 MNm^{-1}
χ	-0.82

Single Harmonic Data

4.1. Model Form Assessment

Parametric Identification

- No differences observed when fitting single harmonic data
- The Dahl model is **unable to fit subcycles satisfactorily**.



Par.	Value
σ_0	20.22 $\mathrm{MNm^{-1}}$
F_c	20.64 N
n	1.02
F_s	18.69 N
K_t	104.08 $\mathrm{MNm^{-1}}$
χ	-0.89

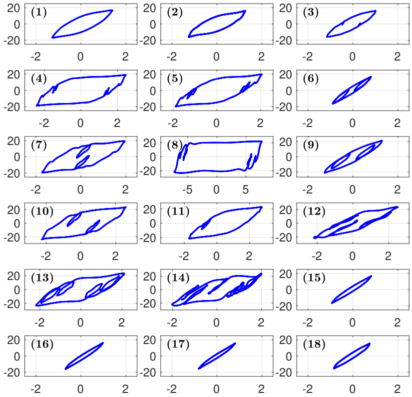
Multi-Harmonic Data (1, 3 harmonics)

The Bouc-Wen model shows a similar performance as the Dahl model.

4.2. Predictive Assessments

Parametric Identification

- Predictive assessments are made with a block-wise approach, considering **the tests before and after the macro-slip event (Test 8)** separately



Force (N) vs Displacement (μm) Measurements

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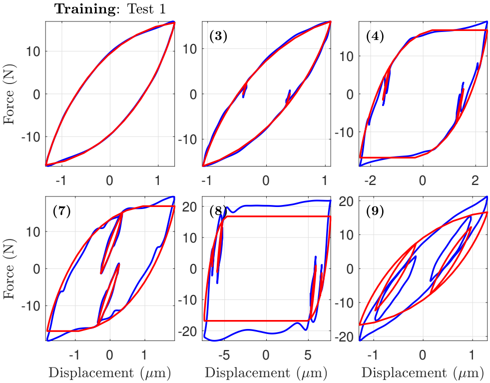

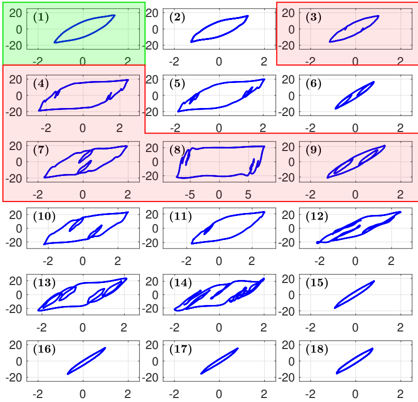

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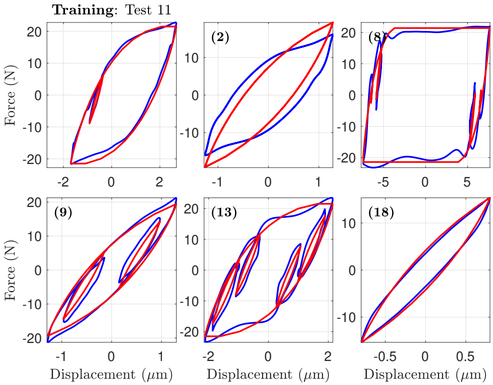
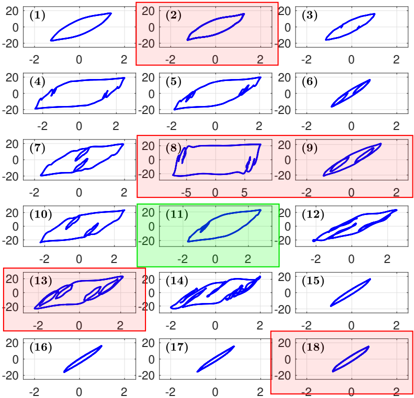
Parametric Identification

- Predictive assessments are made with a block-wise approach, considering **the tests before and after the macro-slip event (Test 8)** separately





Force (N) vs Displacement (μm) Measurements

5. Conclusions

- Experimental observations have been made on the frictional contact of metallic speciments in the **micro-slip regime**
- Special emphasis placed on **validation of Masing hypotheses** in the **multi-harmonic regime**
- A **repeatable** parameter identification procedure for the four-parameter Iwan model is developed based on **backbone reconstruction**

Future Work

- A parametric investigation of normal load dependence to **develop scaling relationships for frictional hysteresis under normal load variations**
- **Epistemic correlation investigations** between hysteresis parameters and surface descriptions (roughness, etc.)

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Thank You!

Brake, Matthew R. W. (2017). *The Mechanics of Jointed Structures*. New York, NY: Springer Science+Business Media.
Mathis, Allen T. et al. (July 2020). "A Review of Damping Models for Structures With Mechanical Joints". In: *Applied Mechanics Reviews* 72.4.
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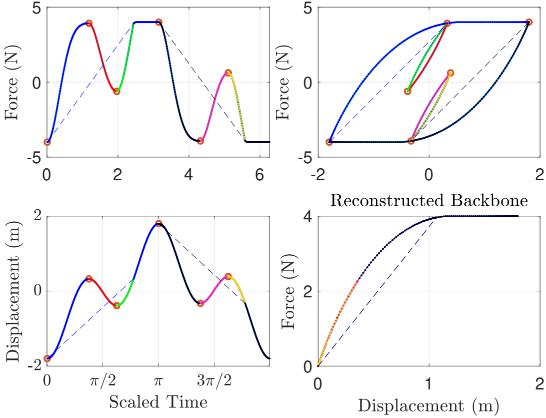
7. Backup Slides

7.1. Backbone Reconstruction from Multiharmonic Data

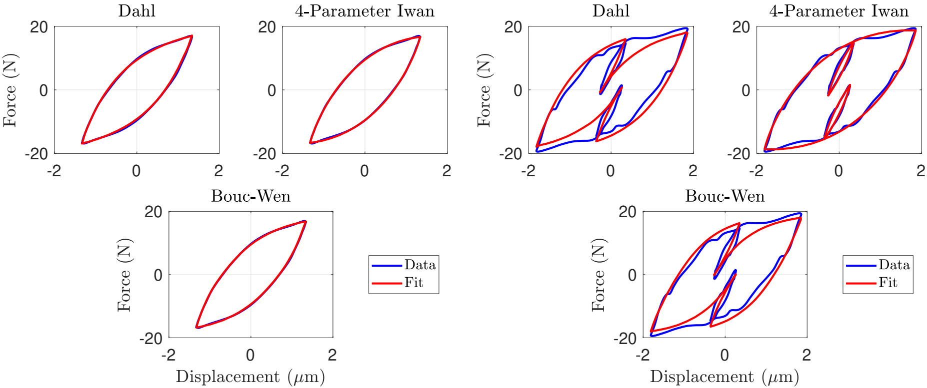
Backup Slides

Goal: Obtain relationship between incremental displacement and force, maintaining history-dependence.



7.2. Bouc-Wen Fitting Performance

Backup Slides



Single Harmonic Data

Multi-Harmonic Data (1, 3 harmonics)