# General Purpose Nonlinear Modal Analysis Capability in FEAST

**$8^{th}$ National Finite Element Developers'/FEAST Users' Meet**

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# 1. Introduction

**Objectives**

- Make a case for **Nonlinear Modal Analysis** in general purpose FE Software.

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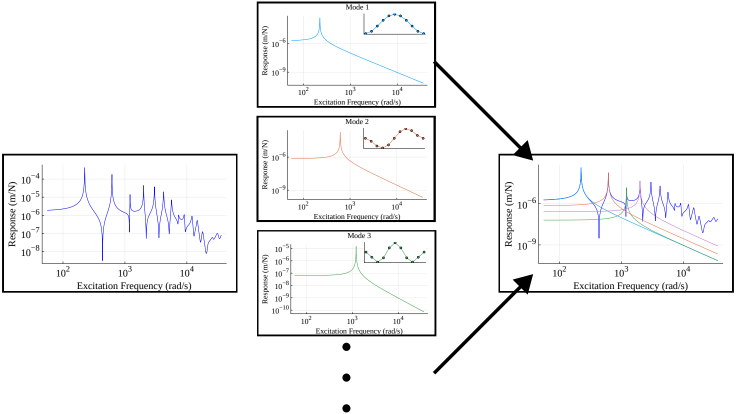
### Outline of the Talk

# 1.1. Why Nonlinear Modal Analysis?

Introduction

- Linear Modal Analysis has been the standard way of dynamical analysis for more than a century now.



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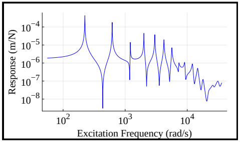
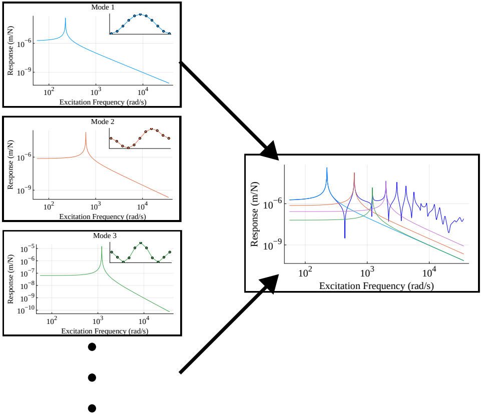
Key Assumptions:

- Linearity
- Amplitude independence.



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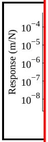
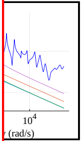
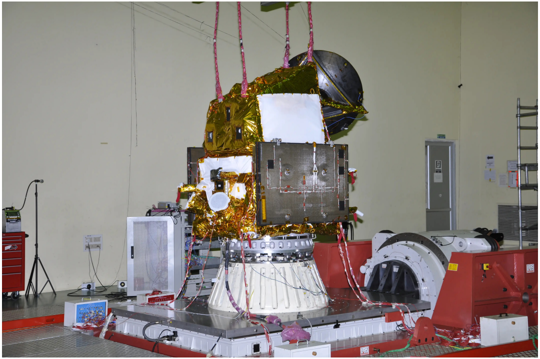



## No real system is just a beam!



*(SCASAT-1 undergoing vibration test Gallery n.d.)*

**Sources of nonlinearity:**

- Joints!
- Materials, environment, etc.

▸ Example

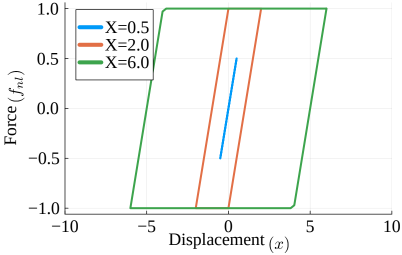
# 1.1. Why Nonlinear Modal Analysis?

Introduction

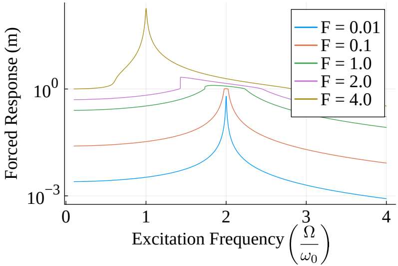
- Let us now consider a simple SDoF oscillator with friction

$$\ddot{x} + c\dot{x} + kx + f_{nl}(x) = F\cos\Omega t.$$

- Let's use the elastic dry friction element to model the friction:




## The Forced Response



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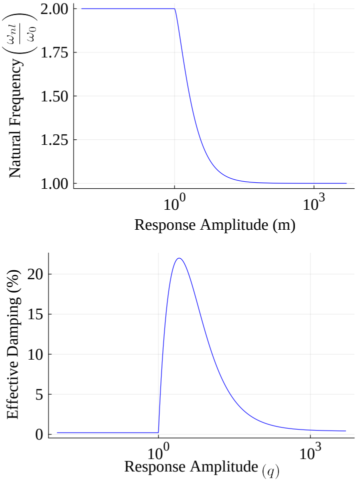
Introduction

## The Nonlinear Modal "Backbones"

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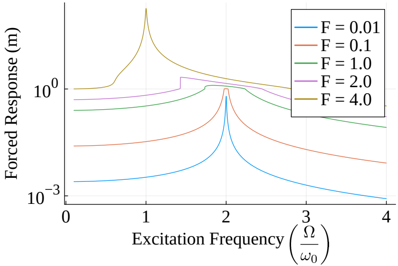

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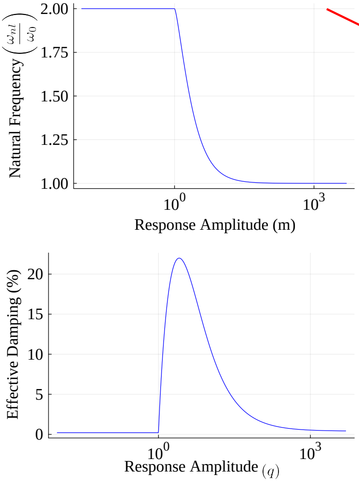


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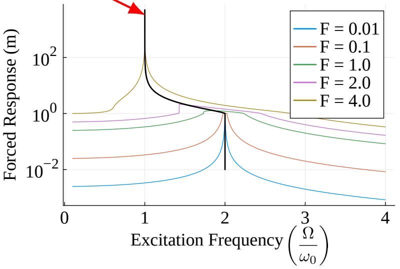


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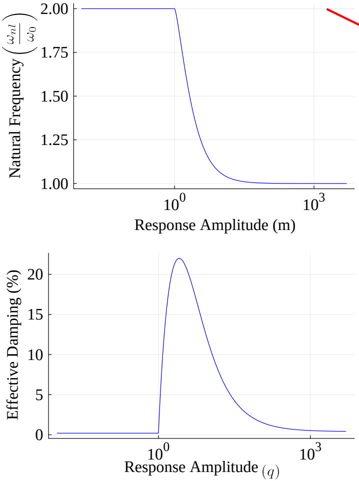


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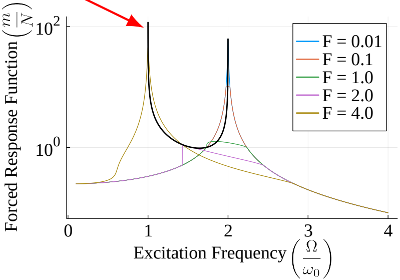


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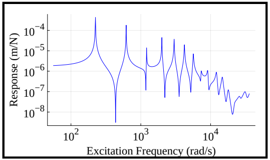
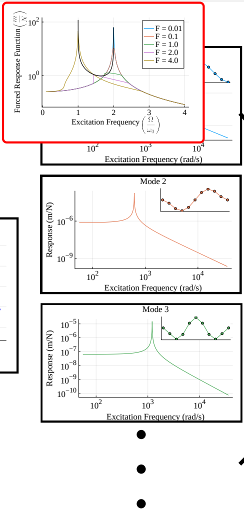
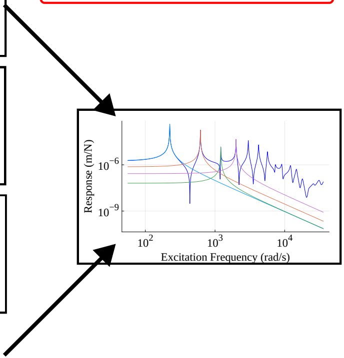
Introduction

Applied NMA employs such backbones in the modal response synthesis



# 2.1. The RQNM Approach

Background on Computational Implementation

- Several computational approaches exist, but quasi-static methods have gained prominence in the assembled structures context (Lacayo and Allen 2019; Balaji and Brake 2020).
- Key here is that the computational effort is identical to a nonlinear static solve.

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**Linear Modal Analysis (LMA)**

For a system described by

$$\underline{\underline{M}}\,\underline{\ddot{u}} + \underline{\underline{K}}\,\underline{u} = \underline{0},$$

LMA involves the solution of

$$\left(\underline{\underline{K}} - \lambda \underline{\underline{M}}\right)\underline{u} = \underline{0} \tag{1a}$$

$$\underline{u}^T \underline{\underline{M}}\,\underline{u} = 1. \tag{1b}$$

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**Courant-Fischer Theorem**

Equation (1b) are the first order optimality conditions of

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### Generalized Rayleigh Quotient Extremization (Balaji and Brake 2020)

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Equation (2b) is only a slight modification of eq. (1b).
$\implies$ Most methods applicable for eq. (1b) can be used.

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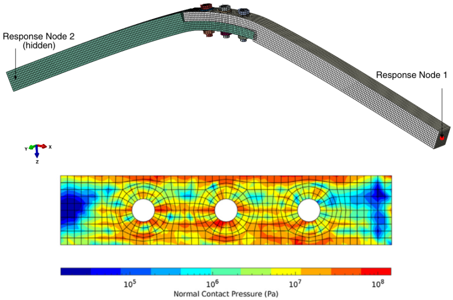
Background on Computational Implementation

- The RQ-NMA process returns amplitude-dependent natural frequency and mode-shape quantities.
- Compared to other NMA techniques, it provides a computationally cheap yet accurate framework.
- Recent studies have shown successful applications to assembled structures.




*Lap-jointed beam (Balaji, Chen, and Brake 2020)*

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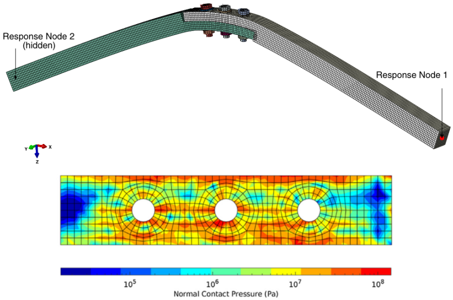


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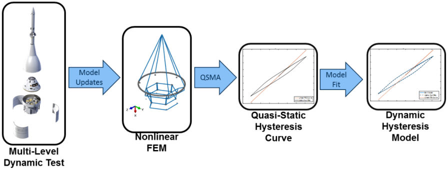
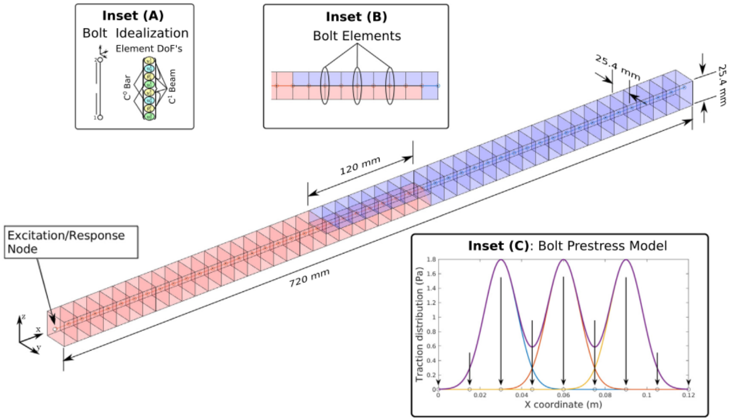



*Orion MPCV (Allen et al. 2021)*

# 3. Numerical Results

## Simplified lap-jointed beam model constructed with 1D beam element




*(Figure from Balaji and Brake 2020)*

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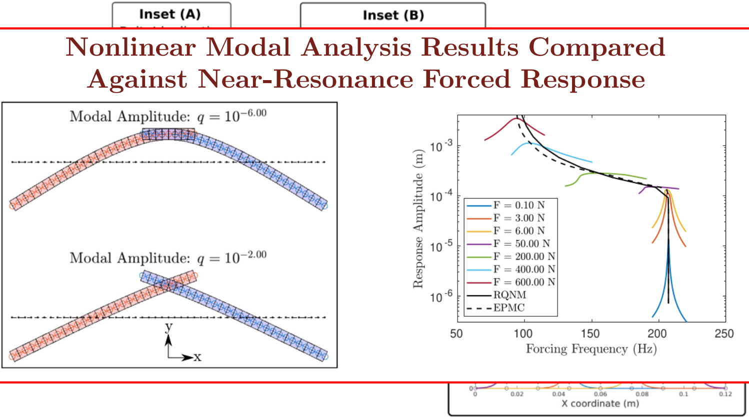
### Nonlinear Modal Analysis Results Compared Against Near-Resonance Forced Response

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Two choices exist:

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2. We develop an approach generalizing Lanczos/Krylov iterations. This may be more computationally efficient

# References I


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[2] R. M. Lacayo and M. S. Allen. "Updating Structural Models Containing Nonlinear Iwan Joints Using Quasi-Static Modal Analysis". Mechanical Systems and Signal Processing, **118**, (Mar. 2019), pp. 133–157. ISSN: 08883270. DOI: 10.1016/j.ymssp.2018.08.034. URL: https://linkinghub.elsevier.com/retrieve/pii/S0888327018305739 (visited on 08/28/2018) (cit. on pp. 13–17).

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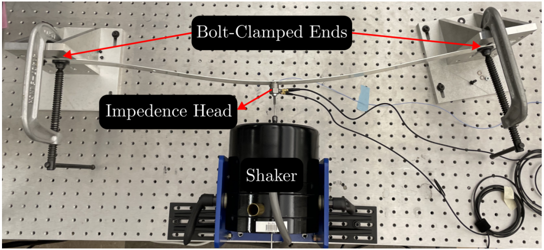
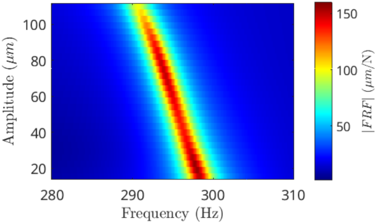
# 6.1. Experimental Observations on a Curved Beam

Backup Slides




*Experimental setup and Forced Response Functions (FRFs) estimated for a curved beam at different mid-point response levels (Figure from Taslicay et al. 2025, Accepted)*

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