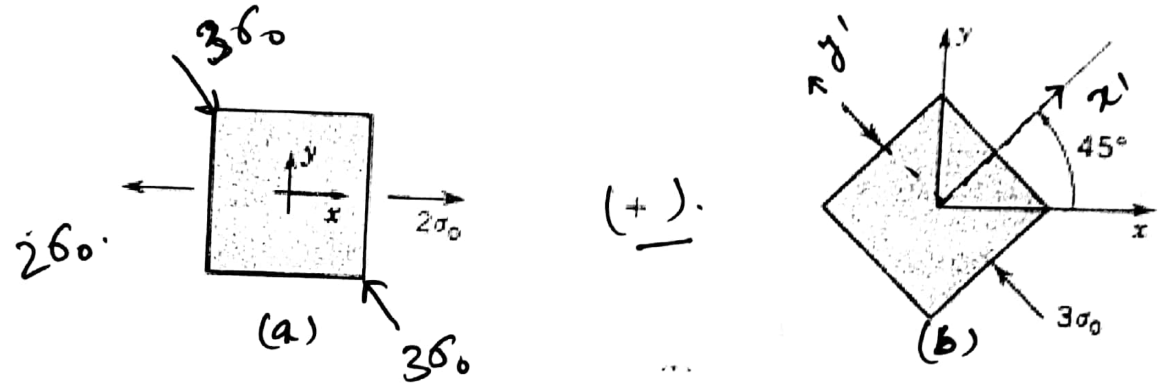
Name: ______ Roll no. ______

Department of Aerospace Engineering, Indian Institute of Technology, Madras.

AS 2010: Basic strength of materials. Quiz 6

1. Find the principal stresses and their directions if the stress at a point is the sum of the two stress states shown:



Transformation equations for plane stress:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

So in diagram (b).

$$\sigma_{x'} = \frac{0 + (-3\sigma_0)}{2} + \frac{0 + 3\sigma_0}{2}\cos 90 + 0 \cdot \sin 90 \Big|_{\theta = 45^\circ}$$

$$\boxed{\sigma_{x'} = -3\sigma_0/2}$$

$$\sigma_{y'} = \frac{0 - 3\sigma_0}{2} - \frac{0 + 3\sigma_0}{2}\cos 90 - 0 \cdot \sin 90 \Big|_{\theta = 45^\circ}$$

$$\boxed{\sigma_{y'} = -3\sigma_0/2}$$ &.

$$\boxed{\tau_{x'y'} = +\frac{3\sigma_0}{2}\sin 90 = 3\sigma_0/2}$$

Superposition of (a) & (b)

$$\sigma_x \Big|_{a\&b} = 2\sigma_0 - 3\sigma_0/2 = \sigma_0/2$$

$$\sigma_y \Big|_{a\&b} = 0 + (-3\sigma_0/2) = -3\sigma_0/2$$

$$\tau_{xy} \Big|_{a\&b} = 3\sigma_0/2$$

- principal angles associated with the principal stresses

$$\tau_{xy} = 0; \quad \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\boxed{\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$$

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2 \times 3\sigma_0/2}{\frac{\sigma_0}{2} - \left(-\frac{3\sigma_0}{2}\right)}\right) = 28.16^\circ$$

$$\boxed{\theta_p = 28.16^\circ}$$

$$\sigma_{1,2} = \frac{\sigma_0/2 + (-3\sigma_0/2)}{2} \pm \sqrt{\left(\frac{\frac{\sigma_0}{2} + \frac{3\sigma_0}{2}}{2}\right)^2 + \left(\frac{3\sigma_0}{2}\right)^2}$$

$$= -\frac{\sigma_0}{2} \pm \sqrt{\frac{13\sigma_0^2}{4}} = \frac{\sigma_0}{2}\left(\pm\sqrt{13} - 1\right)$$

$$\boxed{\sigma_1 = \frac{\sigma_0}{2}\left(\sqrt{13} - 1\right)}$$

$$\boxed{\sigma_2 = -\frac{\sigma_0}{2}\left(\sqrt{13} + 1\right)}$$

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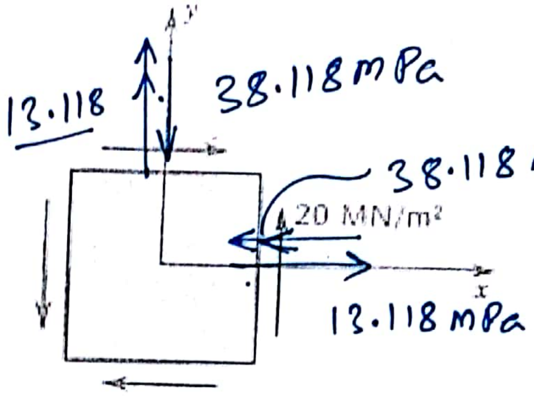
2. At a point in a body in plane stress, the shear stress state is as sketched. Also, the principal stresses are 20 MPa and −45 MPa. Find σ_{xx}, and σ_{yy} and mark them in the figure below.

↑(1) , ↑(2)

Given

$\sigma_1 = 20$ MPa

$\sigma_2 = -45$ MPa



$\tau_{xy} = 20$ MPa

we know that

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

so

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 20^2} = 20 \quad \text{— (i)}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 20^2} = -45^\circ \quad \text{— (ii)}$$

Solving equation (i) & (ii), we get

$$\sigma_x - \sigma_y = \pm 51.235 \quad \text{— (iii)}$$

$$\sigma_x + \sigma_y = -25 \quad \text{— (iv)}$$

from (iii) & (iv)

$\sigma_x = -38.118$ (C) & 13.118 MPa

$\sigma_y = 13.118$ & −38.118 MPa

So; $(\sigma_x, \sigma_y)_1 = (13.118, -38.118)$ &

$(\sigma_x, \sigma_y)_2 = (-38.118, 13.118)$