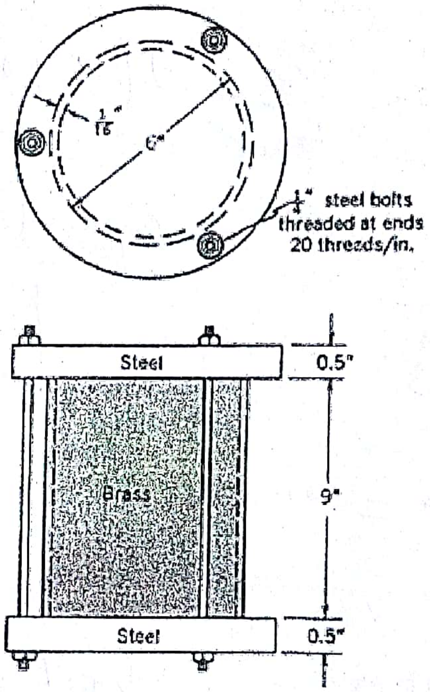
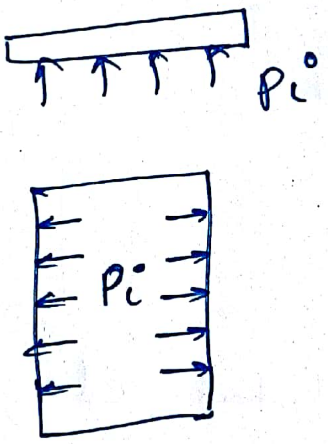
Name: ____________________ Roll no. __________

Department of Aerospace Engineering, Indian Institute of Technology, Madras.

AS 2010: Basic strength of materials. Quiz 8

1. A small experimental pressure vessel is made from a 9 inch long brass cylinder ($E = 15 \times 10^6$ psi [pounds per square inch]) of 6 inch mean diameter, and 1/16 inch wall thickness, and two 1/2 inch thick steel plates held together by three 1/4 inch diameter steel bolts set on a 7 inch diameter bolt circle. The vessel is put together with nuts on the three bolts brought up snug. Then, each nut is tightened one-half turn additional. Estimate the internal pressure in psi at which the vessel is certain to leak.





No. of bolts - 3

Total axial load = $P_i \times \frac{\pi}{4}(d_m)^2$ ← mean dia.

$= \frac{\pi}{4} \times 36 \times P_i$

Force acting in each bolt (F_b) = $\frac{\text{Total axial load}}{3} = 3\pi P_i$

Stress on bolt = $\sigma_b = \frac{F_b}{A_{bolt}}$ ~~Abolt~~ $= \frac{F_b}{A_{steel}}$

$\boxed{\cancel{\sigma_b =} \; \cancel{\frac{3\pi P_i}{A_{steel}}}}$ $\sigma_b = \frac{3\pi P_i}{\pi/(\frac{1}{4})^2}$

Total elongation of bolt $(\delta_b) = \varepsilon_b \cdot L_{bolt}$

Length of bolt = 9 + 0.5 + 0.5 = 10"

$$\varepsilon_b = \frac{\sigma_b}{E_{steel}} = \frac{3\pi P_i}{\frac{\pi}{4}(1/4)^2 \times E_{steel}} = \frac{192 P_i}{E_{steel}}$$

$$\boxed{\delta_b = \frac{192 P_i}{E_{steel}} \times 10''}$$

Because of poisson effect, brass cylinder will contract.

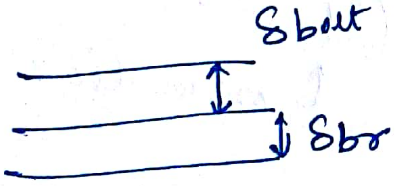
$$\varepsilon\cancel{\varepsilon}_{br} = \frac{\sigma_z}{E_{br}} - \nu\left(\frac{\sigma_\theta + \sigma_r}{E_{br}}\right).$$

Circumferential stress in cylinder $(\sigma_\theta) = \left(\frac{P_i.d}{2t}\right)$.

$$\boxed{\sigma_z = 0;\ \sigma_r = 0}.$$

$$\boxed{\varepsilon_{br} = -\frac{\nu_{br} P_i.d}{2E_{br}t}}$$

Total elongation $= \varepsilon_{br} \cdot L_{br} = \frac{-\nu_{br} P_i d}{2E_{br} \times t} \times 9''$



Compatibility equation

$$\delta_{br} + \delta_{bolt} = \frac{1}{2}\left(\frac{1}{20}\right)''$$

Esteel = 200GPa
Convert into Psi
= 29×10^6 Psi

$$\frac{192 P_i}{29 \times 10^6} \times 10'' + \frac{0.31 \times P_i \times 6}{2 \times 15 \times 10^6 \times (1/16)} \times 9'' = \frac{1}{40}''$$

$$\boxed{P_i = 332.7 \text{ Psi}}.$$

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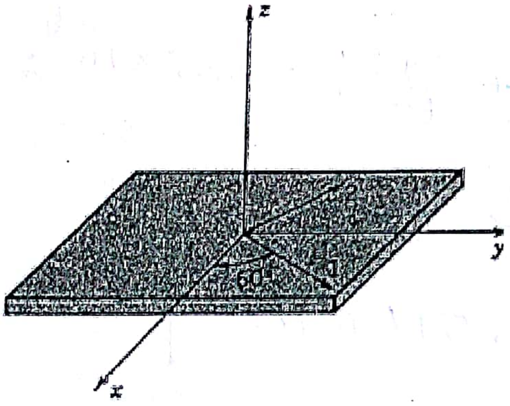
2. The principal strains in the plane of a thin flat aluminium plate (E = 70 GPa, $\nu = 0.3$), which is loaded in its plane are

$$\epsilon_1 = 320 \times 10^{-6}, \text{ and}$$
$$\epsilon_2 = -540 \times 10^{-6}.$$

Find the stresses σ_{xx}, σ_{yy} and σ_{xy}, where the x and y axes are located as shown.



principal strains are given.

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \quad \ell .$$

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right).$$

3 unknown
3 equations.

$$2 \times 60^\circ = \tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right); \quad \gamma_{xy} = \sqrt{3}\,(\varepsilon_y - \varepsilon_x) \text{ — ①}$$

ε_1 ~~&~~ Substitute γ_{xy} value in principal strains equation

$$\varepsilon_1 = \frac{3\varepsilon_x - \varepsilon_y}{2} \quad \& \quad \varepsilon_2 = \frac{3\varepsilon_y - \varepsilon_x}{2}$$

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$$320 \times 10^{-6} = \frac{3\varepsilon_x - \varepsilon_y}{2}$$

$$-540 \times 10^{-6} = \frac{3\varepsilon_y - \varepsilon_x}{2}$$

$\boxed{\varepsilon_x = 105 \times 10^{-6}}$ & $\boxed{\gamma_{xy} = -430\sqrt{3} \times 10^{-6}}$.

$\boxed{\varepsilon_y = -325 \times 10^{-6}}$

$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E}$; $105 \times 10^{-6} = \frac{\sigma_x}{70 \times 10^9} - \frac{0.3}{70 \times 10^9}\sigma_y$ — ⓐ

$\varepsilon_y = -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E}$; $-325 \times 10^{-6} = \frac{-0.3\sigma_x}{70 \times 10^9} + \frac{\sigma_y}{70 \times 10^9}$. — ⓑ

from a & b

$\boxed{\sigma_x = 0.577 \text{ MPa}}$

$\boxed{\sigma_y = -22.577 \text{ MPa}}$

$\tau_{xy} = \gamma_{xy} \times G$.

$= -430 \times \sqrt{3} \times 10^{-6} \times \frac{70 \times 10^9}{2(1+0.3)}$

$\boxed{\tau_{xy} = -20.05 \text{ MPa}}$.