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Lecture 9.

Polarizer: (Lecture 7)

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}}_{P_\theta} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

emergent — incident

$$A' = P_\theta \, A.$$

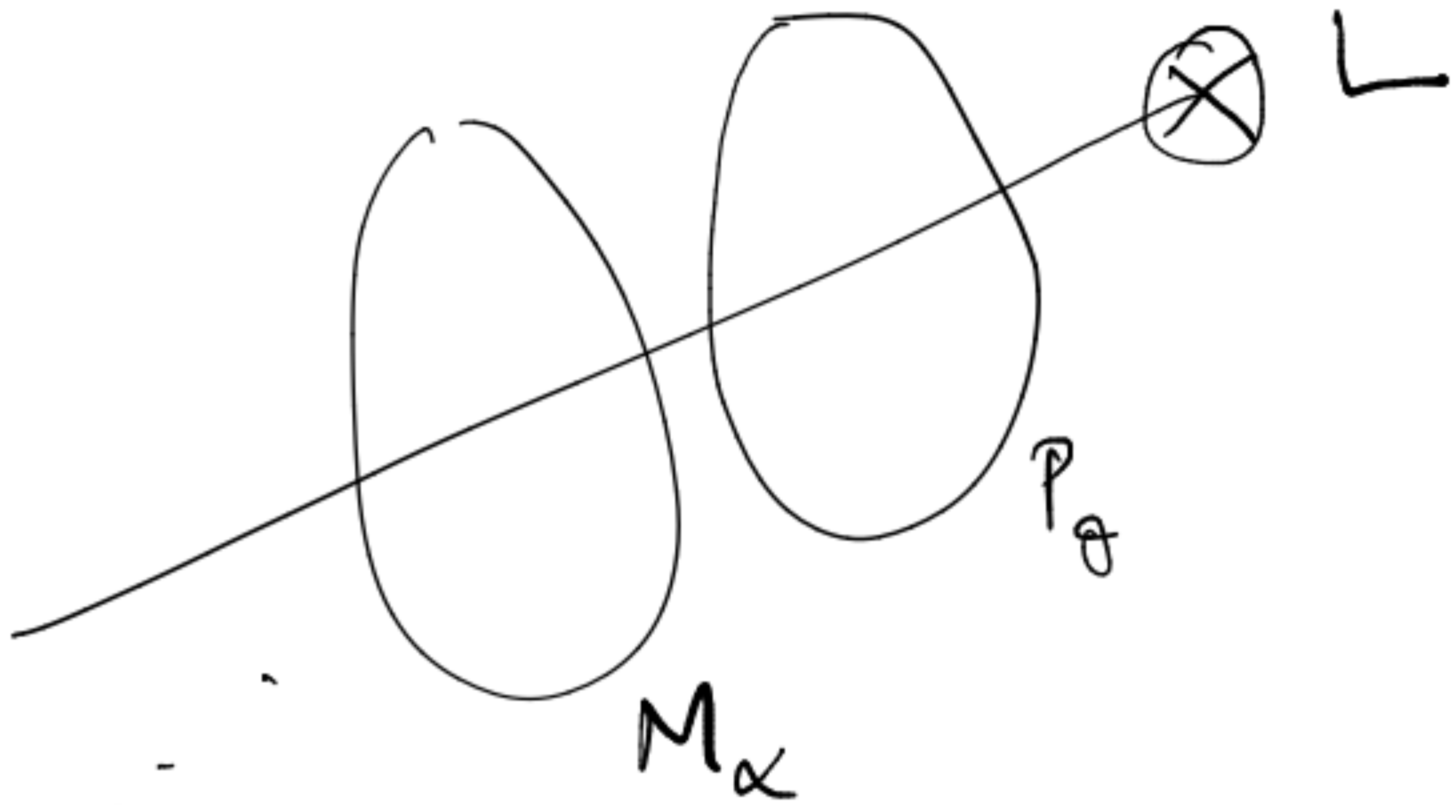
Birefringent plate (Lecture 8)

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \begin{pmatrix} \cos^2\alpha + e^{i\Delta}\sin^2\alpha & (1-e^{i\Delta})\sin\alpha\cos\alpha \\ (1-e^{i\Delta})\sin\alpha\cos\alpha & \sin^2\alpha + e^{i\Delta}\cos^2\alpha \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}.$$

emergent. — incident

$$A' = M_\alpha \cdot A.$$

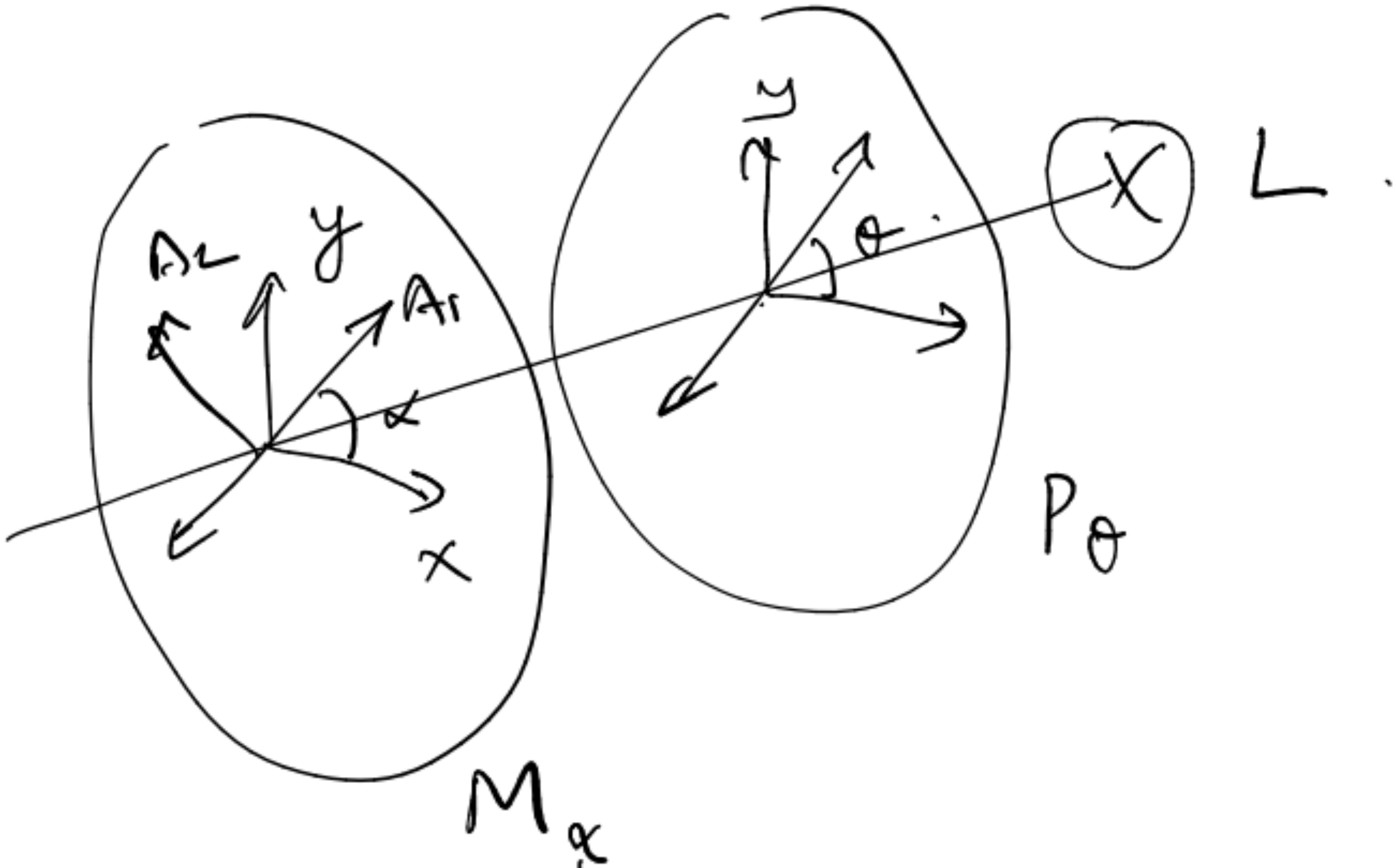
An optical system =



complex amplitudes of light from L.

$$A' = \underbrace{M_\alpha \underbrace{P_\theta A}_{\text{emergent wave from the polariser.}}}_{\text{emergent light from the birefrigent sheet}}$$

Optical system to produce circularly polarised light:



For $\Delta = \frac{\pi}{2}$ (Quarter wave plate) of the Birefringent sheet, we will get circularly polarised light, provided

$$\theta = 0, \quad \alpha \neq \left(0 \text{ or } \frac{\pi}{2}\right)$$

OR

$$\theta = \frac{\pi}{2}, \quad \alpha = \left(0, \text{ or } \frac{\pi}{2}\right).$$

For circularly polarised light, recall.

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = A \begin{pmatrix} 1 \\ \pm i \end{pmatrix} \quad \cdots \quad (1)$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & \mp i \\ \mp i & 1 \end{pmatrix}}_{\text{Quarterwave plate}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$= \begin{pmatrix} \mp i \\ 1 \end{pmatrix} A_y$$

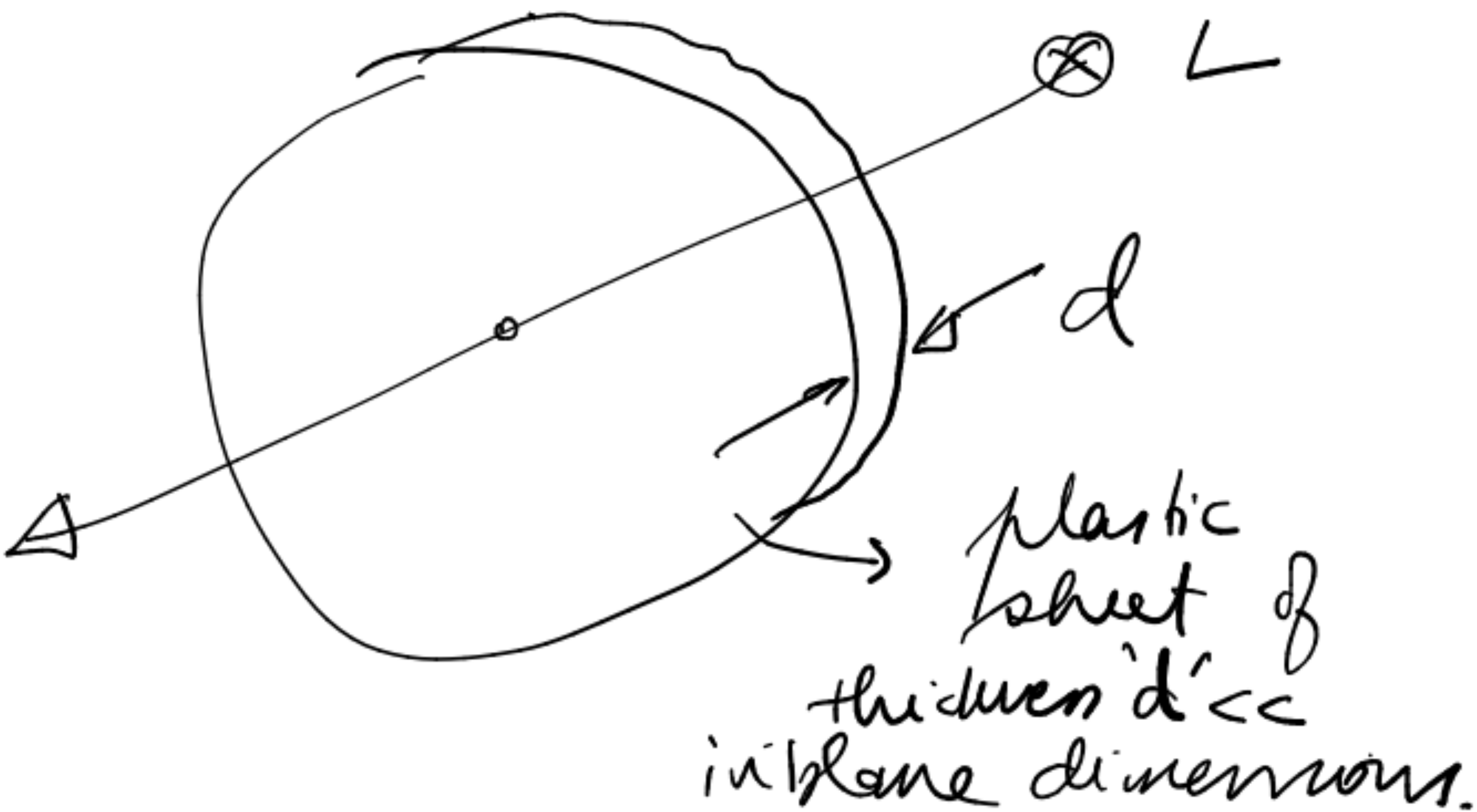
$$= \begin{pmatrix} \mp 1 \\ i \end{pmatrix} A_y \quad \cdots \quad (2)$$

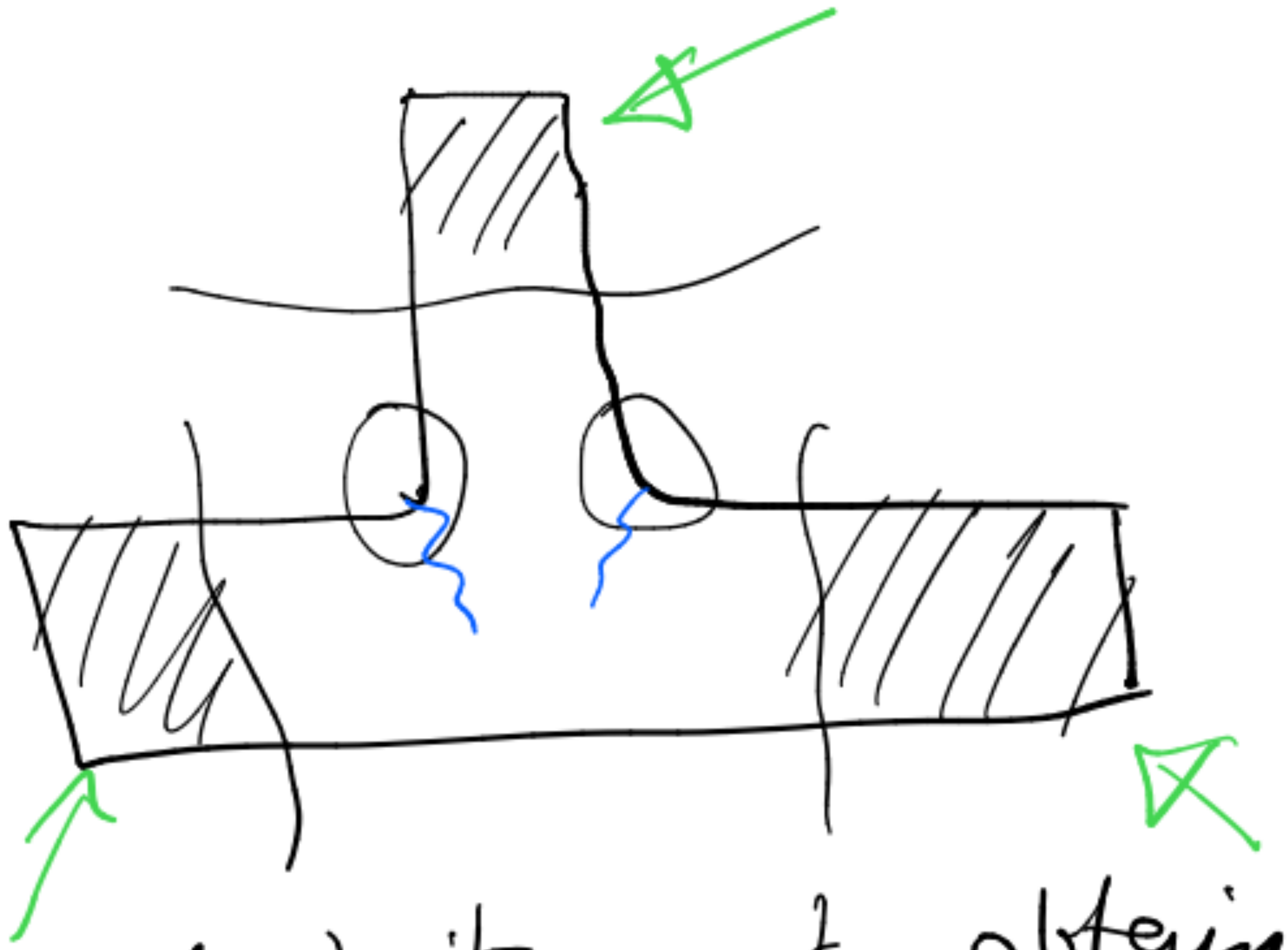
Compare (1) & (2) to see that a circularly polarised light beam emerges from the quarter wave plate.

Elliptically polarised light

→ set $\alpha \neq 0$ or $\frac{\pi}{4}$ or $\frac{\pi}{2}$.

Refractive indices of stressed transparent materials





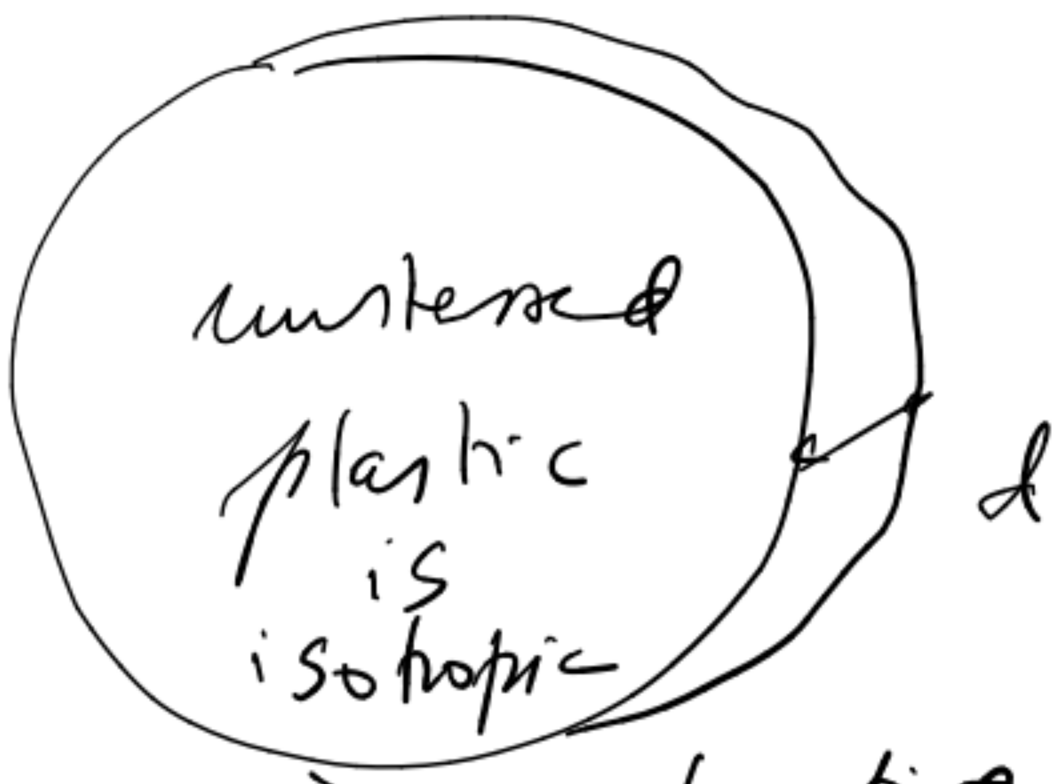
1. Photoelasticity — to obtain critical stresses in non-slender components

2. Photoelasticity is not used for this purpose anymore because FEM has totally replaced it.

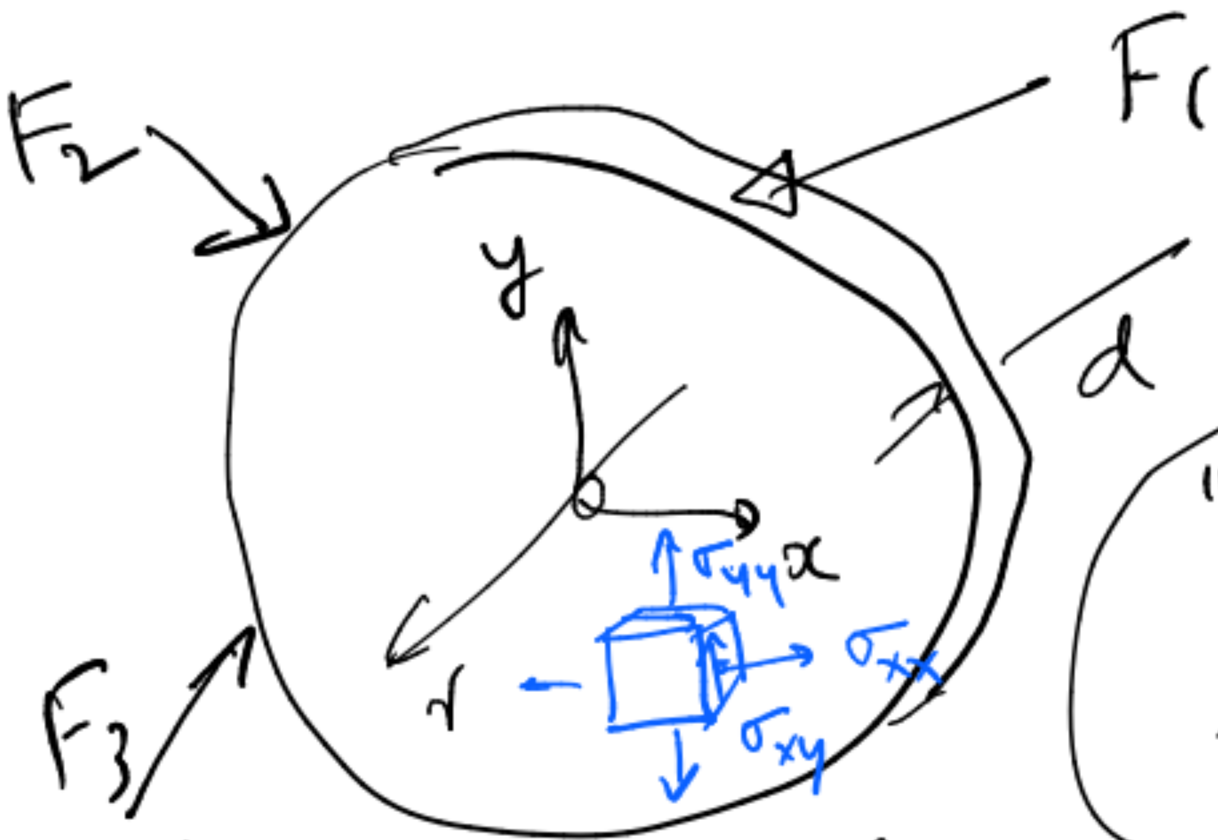
3. When photoelastic analysis was used, a scaled replica of the relevant structure would

be prepared in plastic, loaded, photo elasticity would yield stress values, these values would be scaled to the original component.

How does stress change the refractive indices of thin plastic sheets?



Let n_0 be its refractive index.



"plane stress"

$\sigma_{rr} =$
$\sigma_{rx} =$
$\sigma_{ry} = 0.$

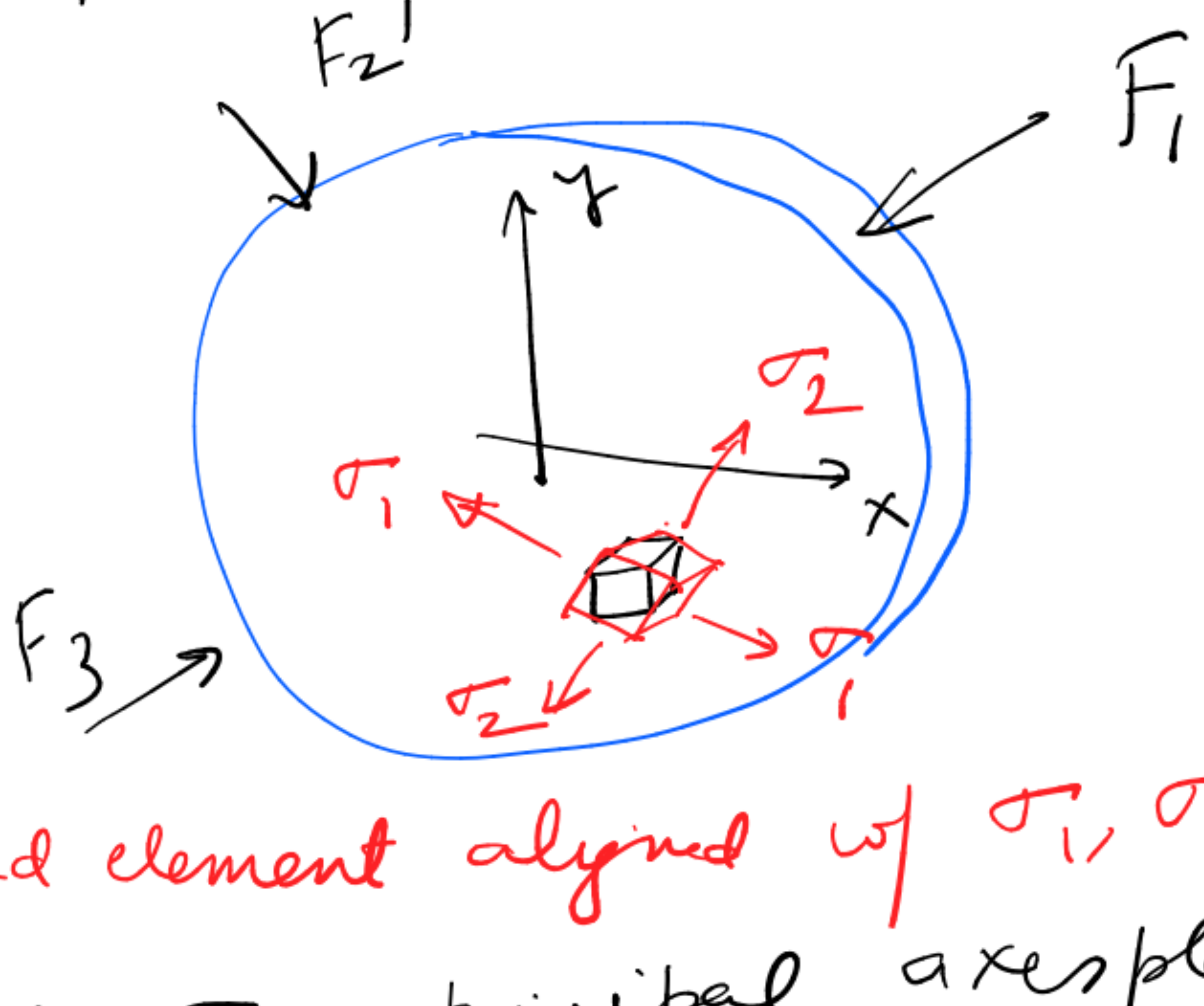
Stressed plastic becomes optically anisotropic.

Optical anisotropy cannot depend on σ_{xx}, σ_{yy}, σ_{xy}

$\sigma_{xx} \neq 0$
$\sigma_{yy} \neq 0$
$\sigma_{xy} \neq 0$

because x- & y- are dependant on the observer & not on the phenomenon of stress generate.

Optical anisotropy must depend only on the principal stresses & the principal directions developed @ each & every pt. in the plate.



red element aligned w/ σ_1, σ_2

σ_1, σ_2 principal axes play the role of A_1, A_2 in the birefringent filter we studied earlier.

The light polarized along σ_1 & σ_2 experience diffent refractive indices.

How ?

Let $\sigma_1, \sigma_2, \sigma_3$ be the 3 principal stresses @ a point.

Then,

Material constat.

$$\text{(a)}\quad n_1 - n_0 = c_1\sigma_1 + c_2(\sigma_2 + \sigma_3)$$

$$\text{(b)}\quad n_2 - n_0 = c_1\sigma_2 + c_2(\sigma_3 + \sigma_1)$$

$$\text{(c)}\quad n_3 - n_0 = c_1\sigma_3 + c_2(\sigma_1 + \sigma_2).$$

n_0 = refractive index of the unstressed state.

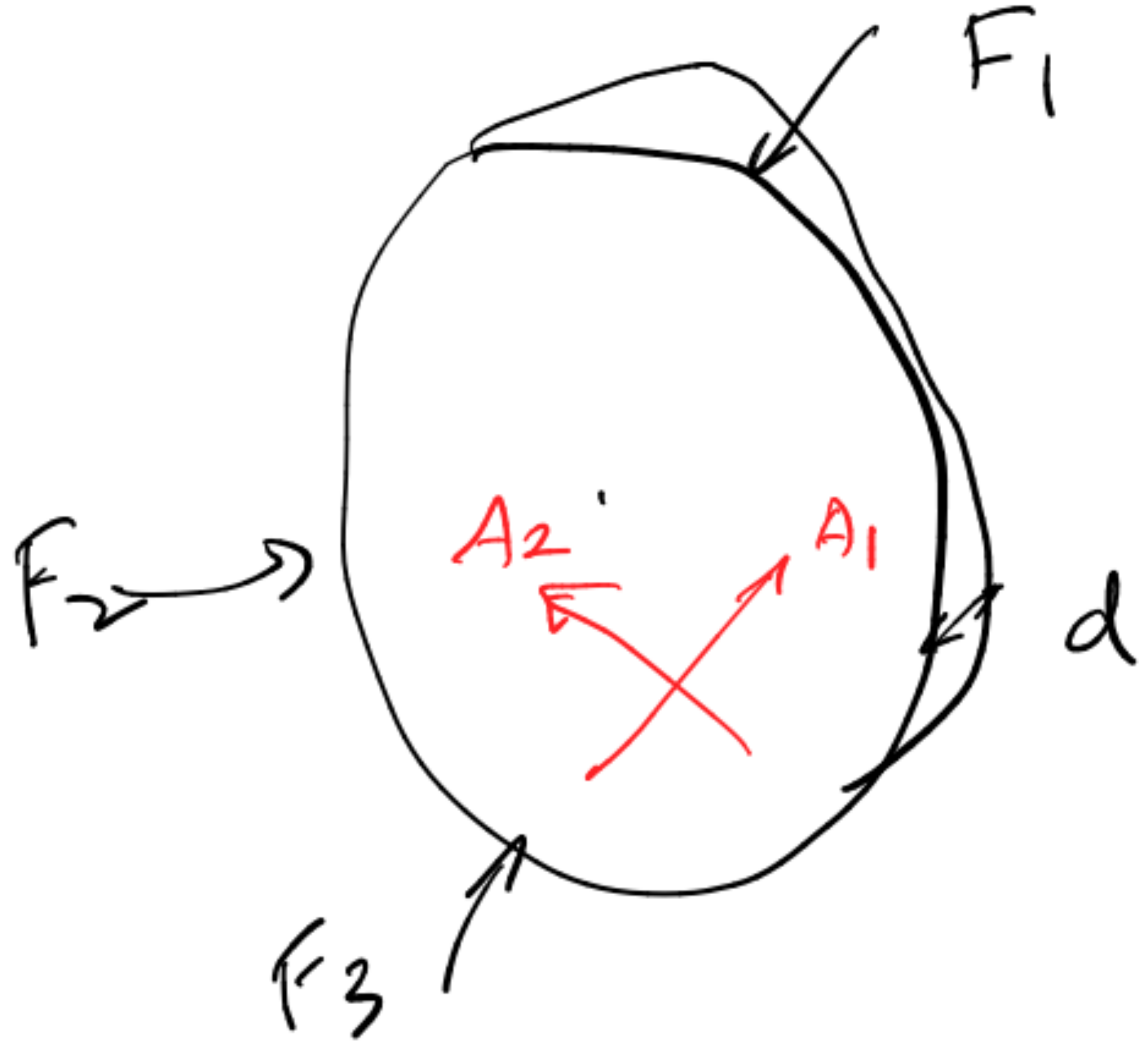
c_1, c_2 = stress-optic coefficients = Material constants.

To eliminate n_0, do

ⓐ − ⓑ, ⓑ − ⓒ, ⓒ − ⓐ :

$$
\boxed{
\begin{aligned}
n_2 - n_1 &= (\sigma_1 - \sigma_2)(c_2 - c_1) \\
n_3 - n_2 &= (\sigma_2 - \sigma_3)(c_2 - c_1) \\
n_1 - n_3 &= (\sigma_3 - \sigma_1)(c_2 - c_1)
\end{aligned}
}
$$

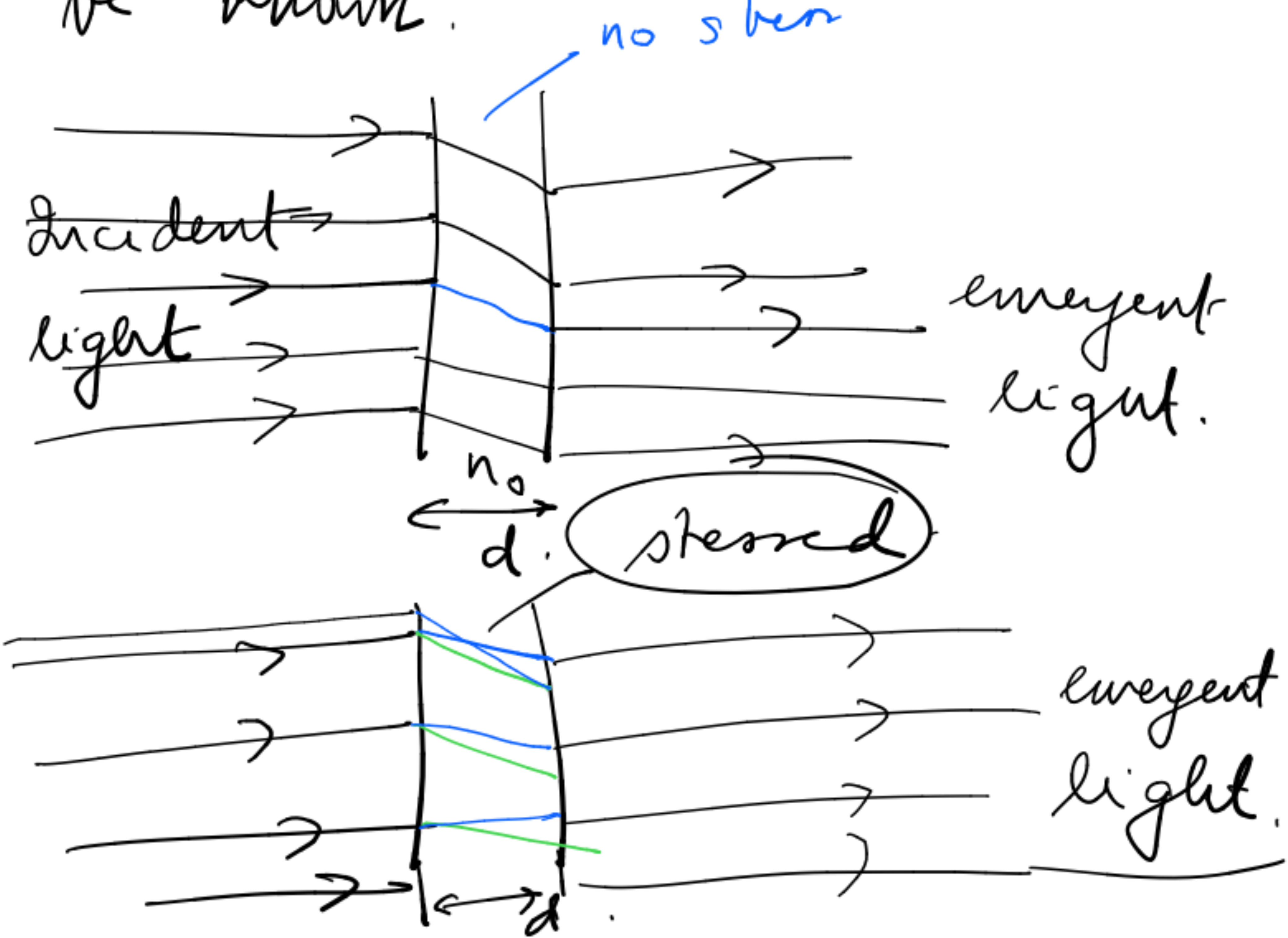
③



polar.

If σ_1, σ_2, & principal directions were known, then α is also known.

But to fully characterize birefringence of the loaded plastic sheet Δ must also be known.



The diffently polarised light beams @ each point travel @ diffent speeds.

$\|^{el}$ $\sigma_1 \longrightarrow v_1$

$\|$ $\sigma_2 \longrightarrow v_2$.

time diffence to travel through the sheet of thicken d =

$$t_2 - t_1 = \frac{d}{v_2} - \frac{d}{v_1}$$

$$= \frac{d}{c}\left\{\frac{c}{v_2} - \frac{c}{v_1}\right\}$$

$$t_2 - t_1 = \frac{d}{c}\left(n_2 - n_1\right)$$

The phase lag corresponding to this time lag is :

$$\Delta = \frac{2\pi}{\lambda} c \left(t_2 - t_1 \right)$$

$$= \frac{2\pi}{\lambda} \cancel{c} \frac{d}{\cancel{c}} \left(n_2 - n_1 \right)$$

$$\boxed{\Delta = \frac{2\pi}{\lambda} d \left(n_2 - n_1 \right)} \quad (4)$$

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Lecture 10

- Incident light is phase shifted by $\Delta = \frac{2\pi}{\lambda} d(n_2 - n_1)$ during passage thru' the loaded artificially biref. plate of thickness d.

- $$n_2 - n_1 = (\sigma_1 - \sigma_2)\, \underset{c_2 - c_1}{\underset{\uparrow}{B}}$$

- B is called relative stress optic coefficient.

B is measured in Brewsters

1 Brewster $= 10^{-6}$ / MPa.

From 8:

$$\sigma_1 - \sigma_2 = f_\sigma \frac{\Delta}{2\pi d} \qquad (7)$$

$$\frac{\lambda}{B}$$

If we can measure Δ, we can find out $\sigma_1 - \sigma_2$.

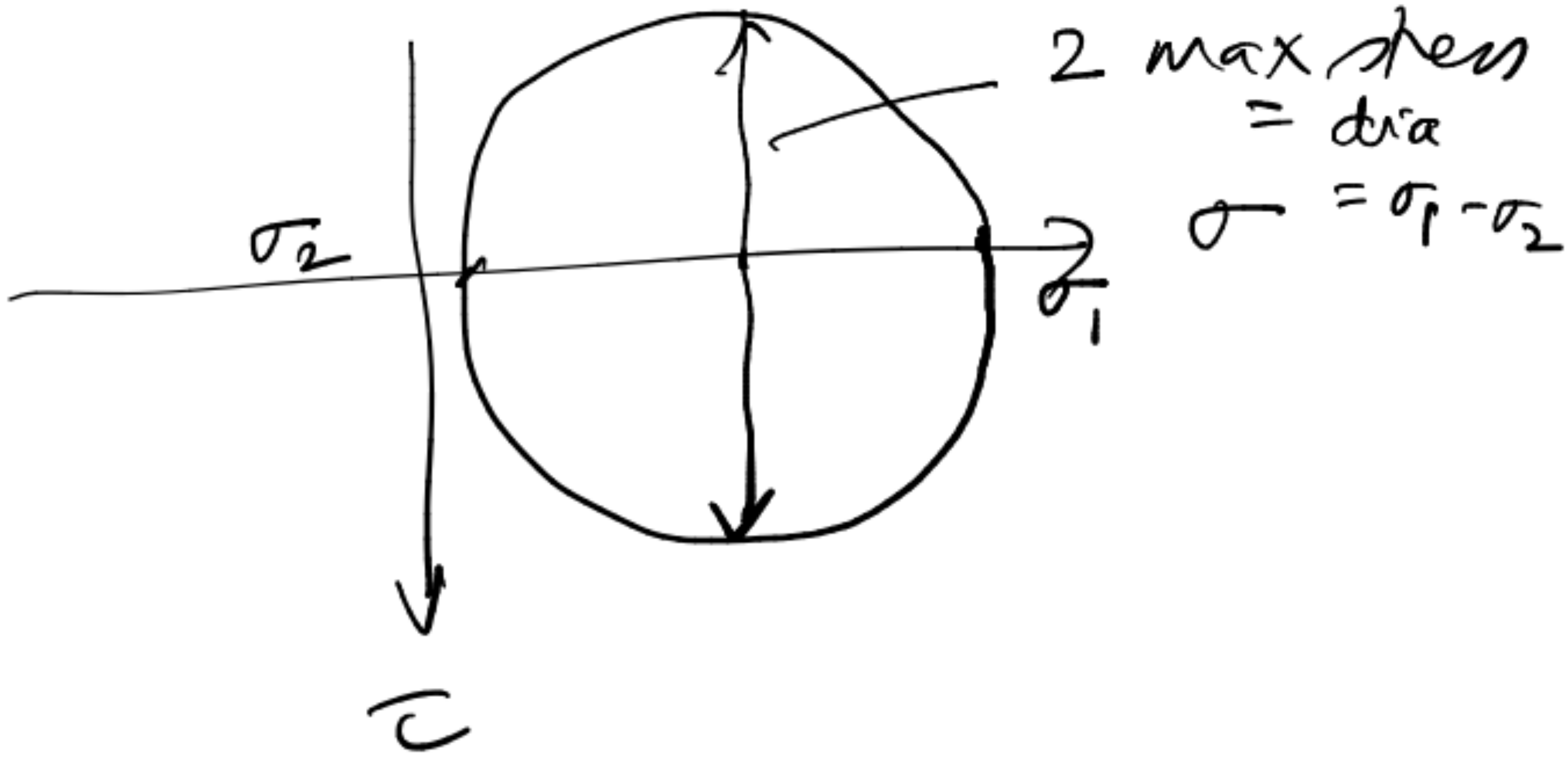
Using Hooke's law for plane σ,

$$\varepsilon_1 = \frac{1}{E}\left(\sigma_1 - \nu\sigma_2\right)$$

$$\varepsilon_2 = \frac{1}{E}\left(\sigma_2 - \nu\sigma_1\right)$$

$$\varepsilon_1 - \varepsilon_2 = \frac{1+\nu}{E}\left(\sigma_1 - \sigma_2\right) \overset{(7)}{=} f_\varepsilon \frac{\Delta}{2\pi d},$$ where

$$f_\varepsilon = \frac{1+\nu}{E} f_\sigma.$$



$\sigma_1 - \sigma_2 = 2 \times$ max shear stress developed @ the point of interest.

failure theories Tresca, VMises use the max. shear stress.

Polariscope: Tool to measure Δ.

Circular polariscope:

Works w/ circularly polarised light.

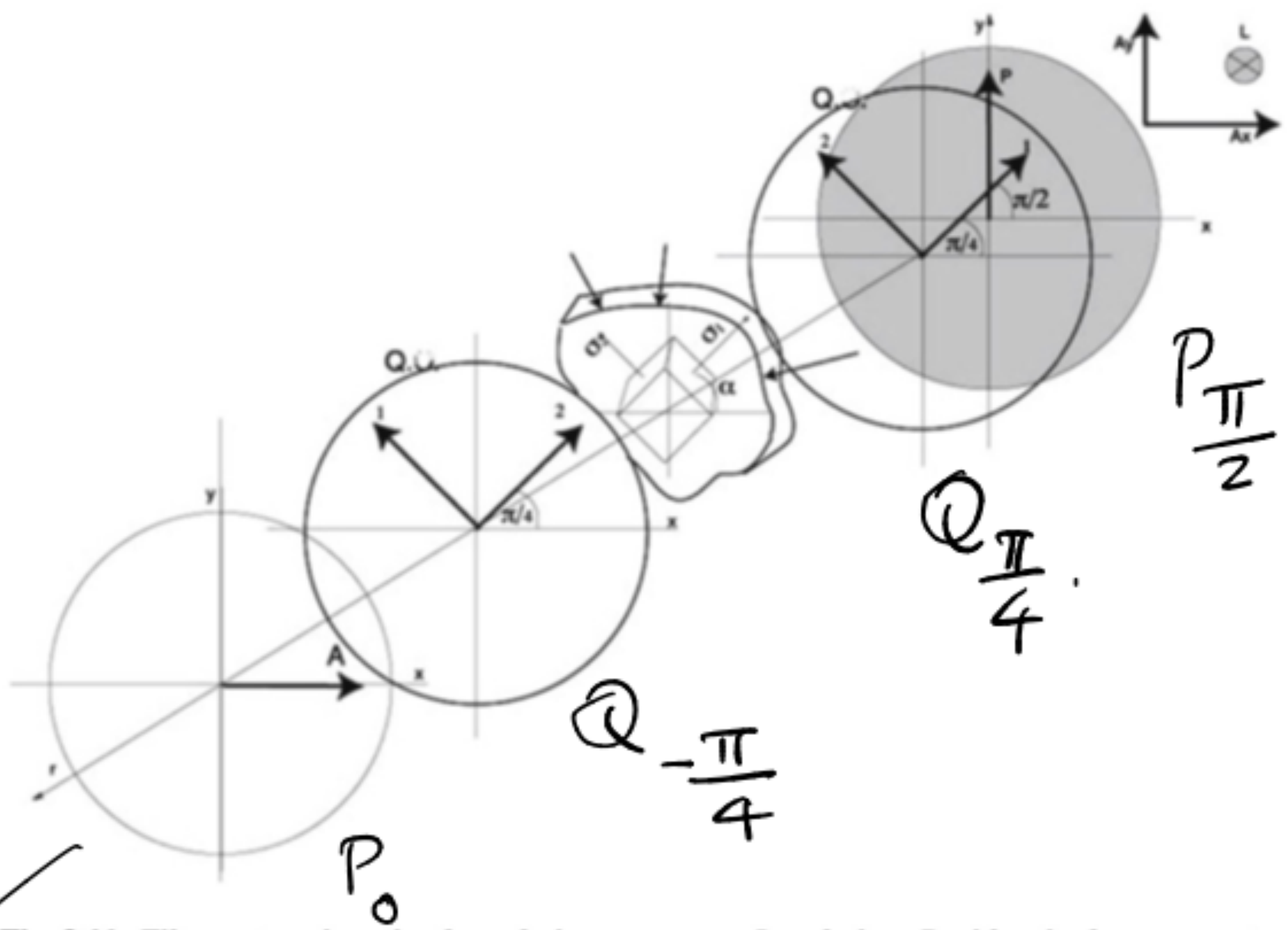


Fig. 3.11 Filters set-up in a circular polariscope: *source L*, polarizer P with axis along y, quarter-wave plate Q with axes 1 and 2 at $\pi/4$ and $3/4\pi$ respectively, plane model, second quarter-wave Q with axes 2 and 1 at $\pi/4$ and $3/4\pi$ respectively, analyzer A with its axis along x. Positive direction counterclockwise looking towards the light-source

photographic film

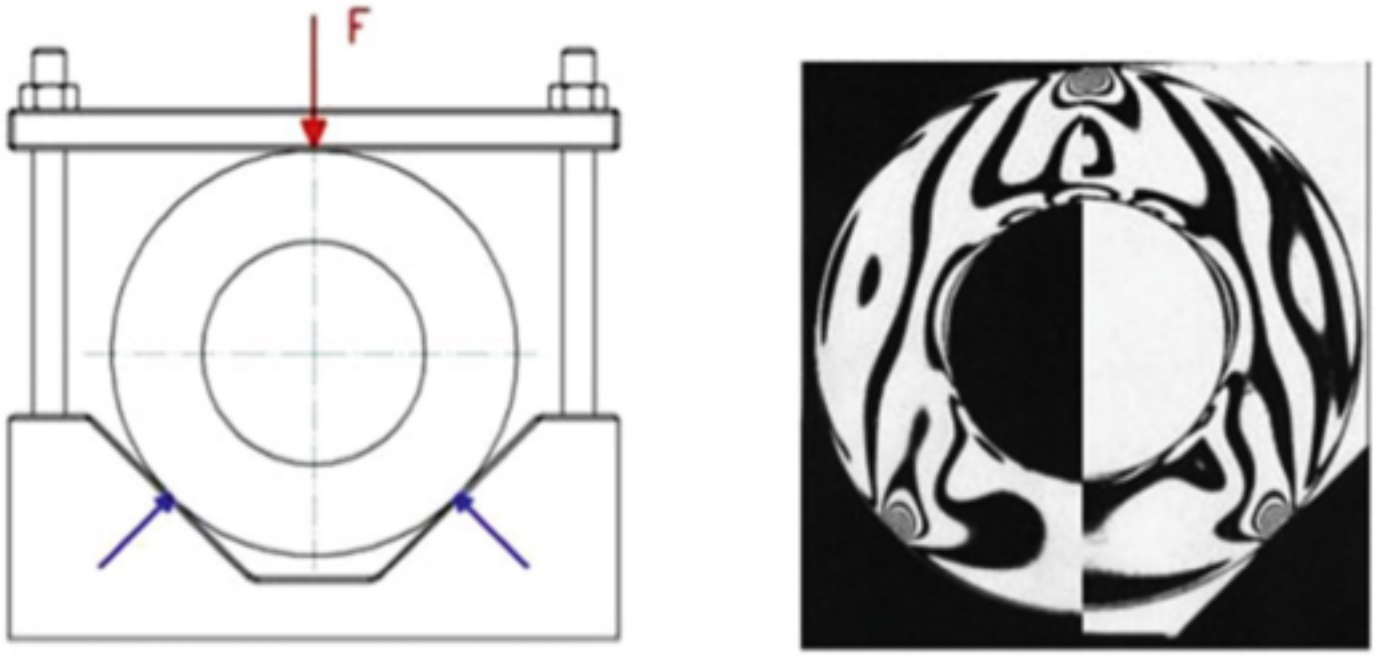


Fig. 3.12 Isochromatics on a black and bright background of an annular disc loaded at three points

Let the wave from L $\otimes$ have complex amplitudes $\begin{pmatrix} A_x \\ A_y \end{pmatrix}$.

$$\begin{pmatrix} A_x \\ A_y' \end{pmatrix} = P_0 \; Q_{-\frac{\pi}{4}} \; M \; Q_{\frac{\pi}{4}} \; P_{\frac{\pi}{2}} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$A'_x = A_y\left(2m_{21} - i\,m_{11} + i\,m_{22}\right).$$

We know from lecture 8

$$\left\{\begin{aligned} m_{11} &= \cos^2\alpha + e^{i\Delta}\sin^2\alpha \\ m_{22} &= \sin^2\alpha + e^{i\Delta}\cos^2\alpha \\ m_{12} &= m_{21} = (1-e^{i\Delta})\sin\alpha\cos\alpha. \end{aligned}\right.$$

$$A'_x = A_y\left(1-e^{i\Delta}\right)\left(\sin 2\alpha - i\cos 2\alpha\right).$$

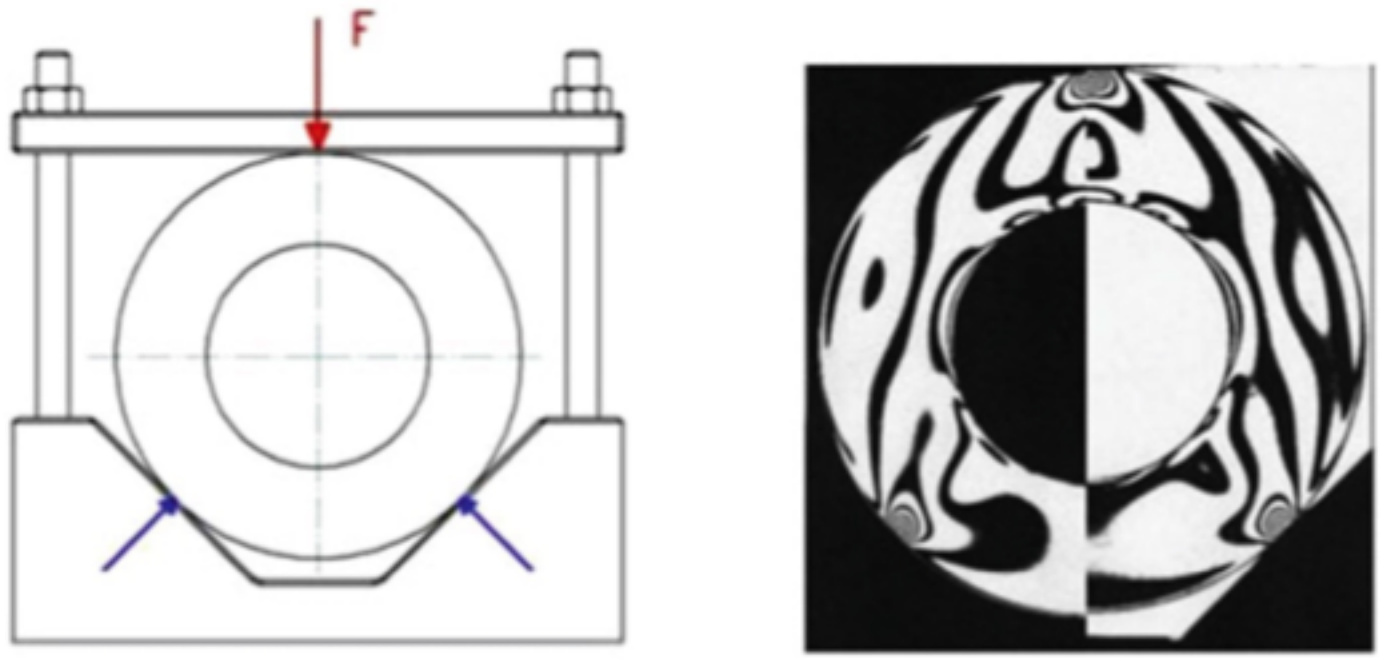


Fig. 3.12 Isochromatics on a black and bright background of an annular disc loaded at three points

In the regions that go dark in the film (dark fringes),

$A'_x = 0.$

Monochromatic illumination.

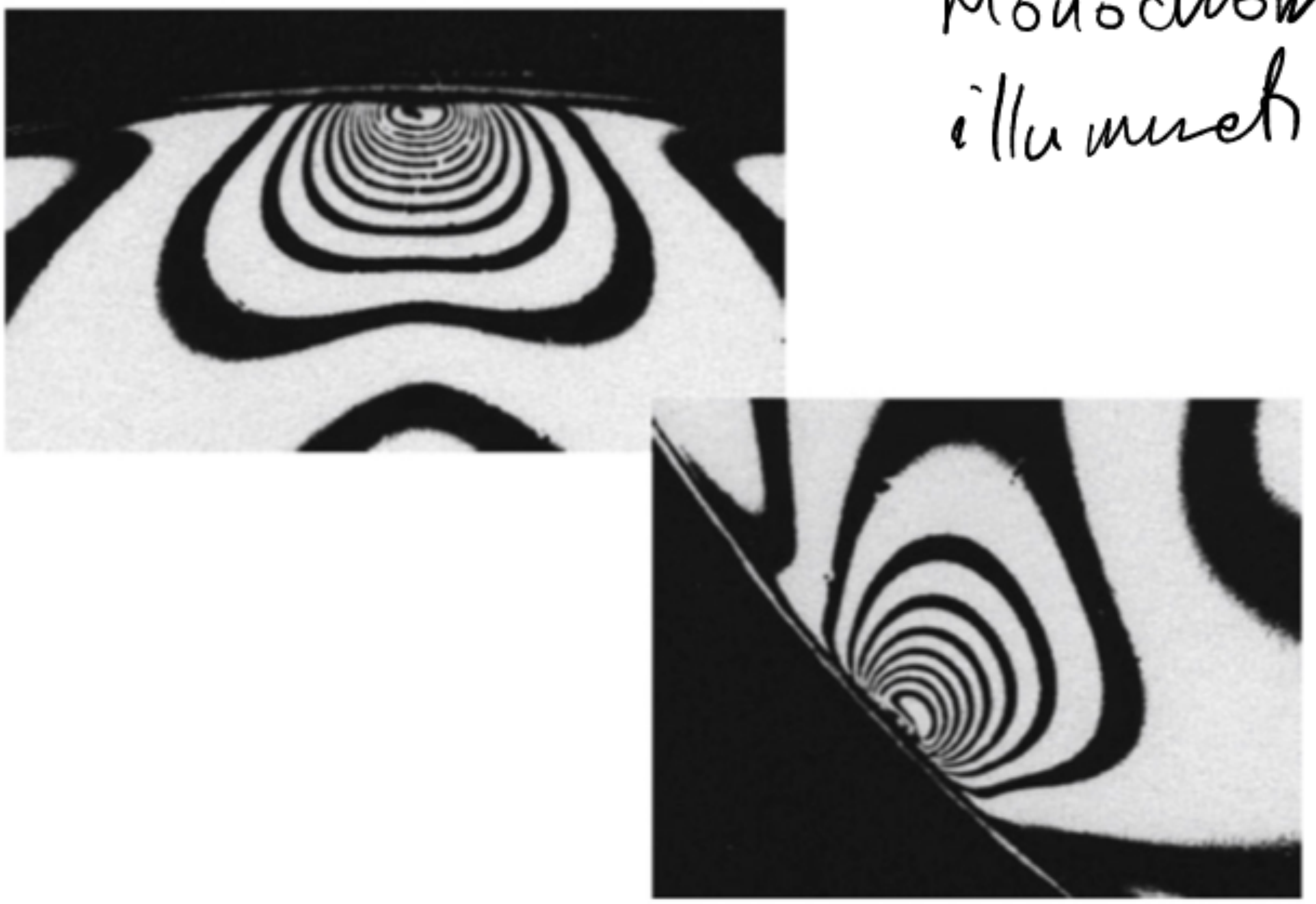
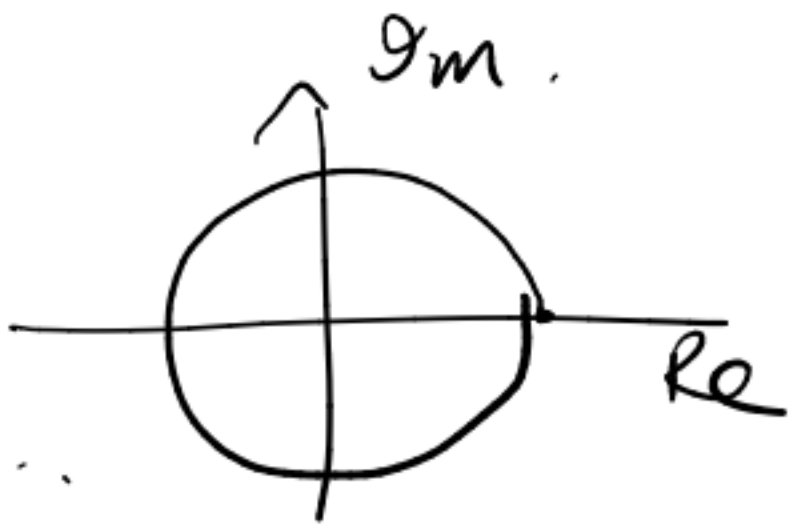


Fig. 3.13 Details of isochromatics in contact points without (*up*) and with a light friction (*bottom*)

Because $\sin 2\alpha - i \cos 2\alpha \neq 0$ for any α, it follows that $1 - e^{i\Delta} = 0$ "extinction condition".

$$\Rightarrow \quad e^{i\Delta} = 1.$$



Solution $\ni \dfrac{\Delta}{2\pi} = 0, 1, 2, \ldots$

If white light were used instead of monochromatic light:

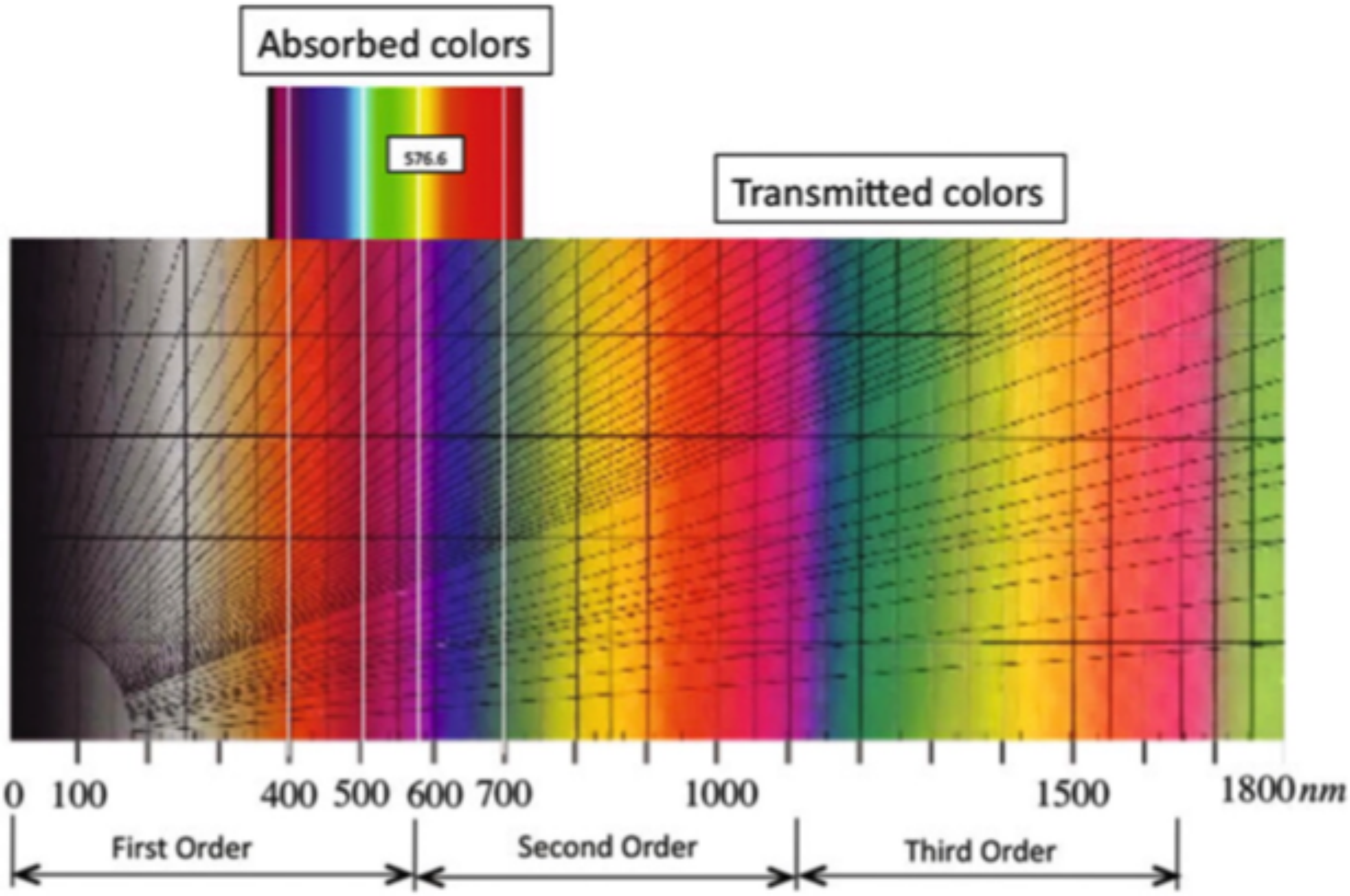


Fig. 3.14 Absorbed and transmitted color: modified from [16]

The colours that appear in the fringe pattern are those that were transmitted. Their complementary color was extinguished. → Use this to find λ & Δ.