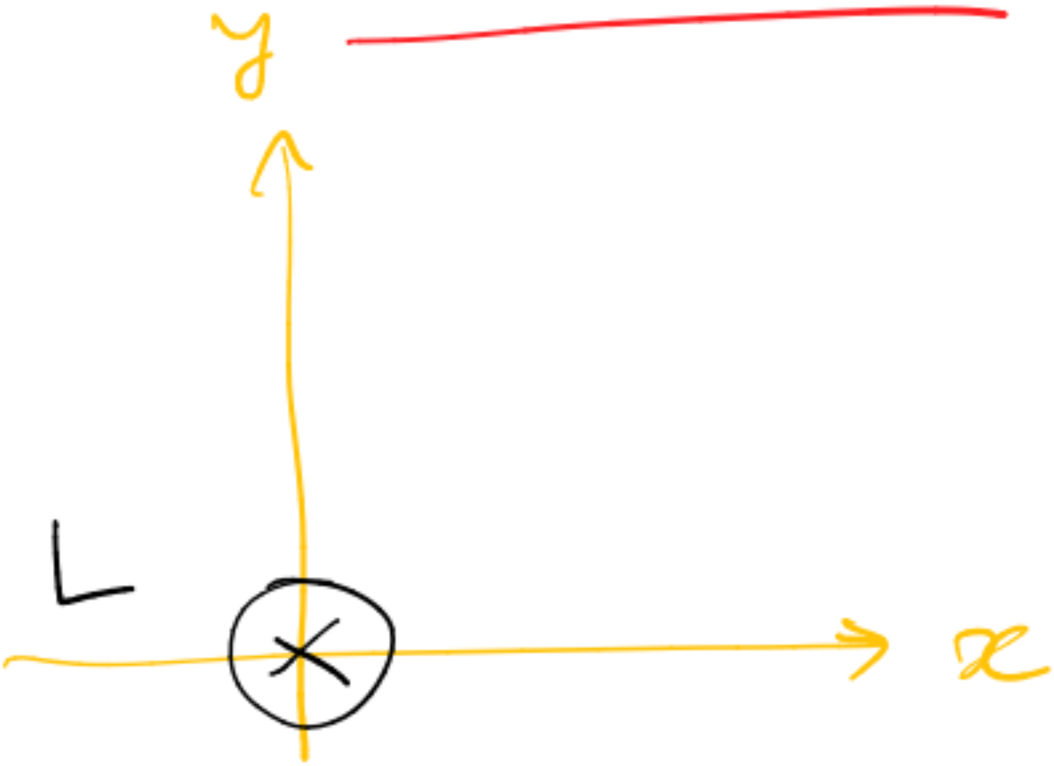
AS3520

Lecture 7



Consider a light wave whose electric field amplitudes along the x- & y- dirns are

$$\begin{pmatrix} A_x e^{i\phi_x} \\ A_y e^{i\phi_y} \end{pmatrix}$$

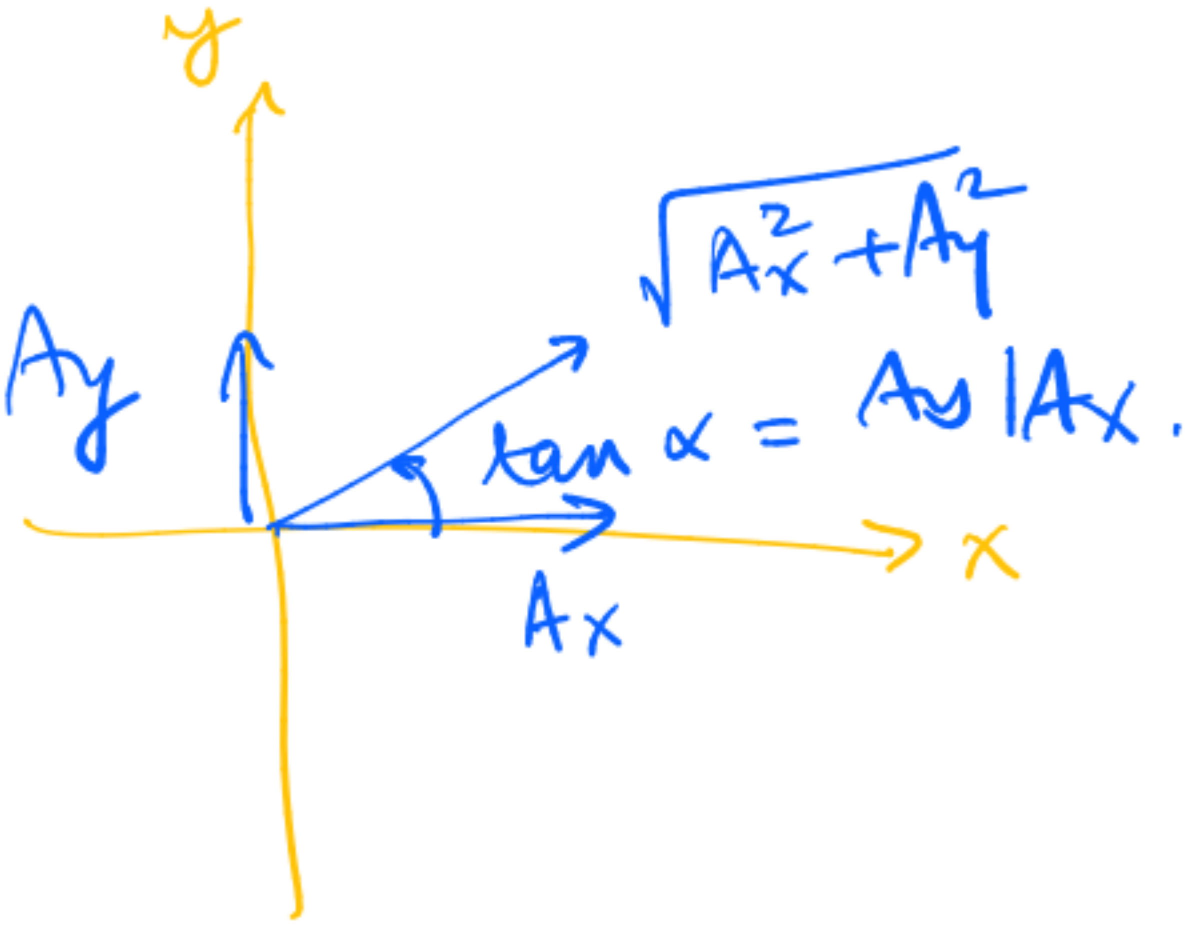
$$= \begin{pmatrix} A_x \\ A_y e^{+i\Delta} \end{pmatrix} e^{i\beta/2} e^{-i\Delta/2} .$$

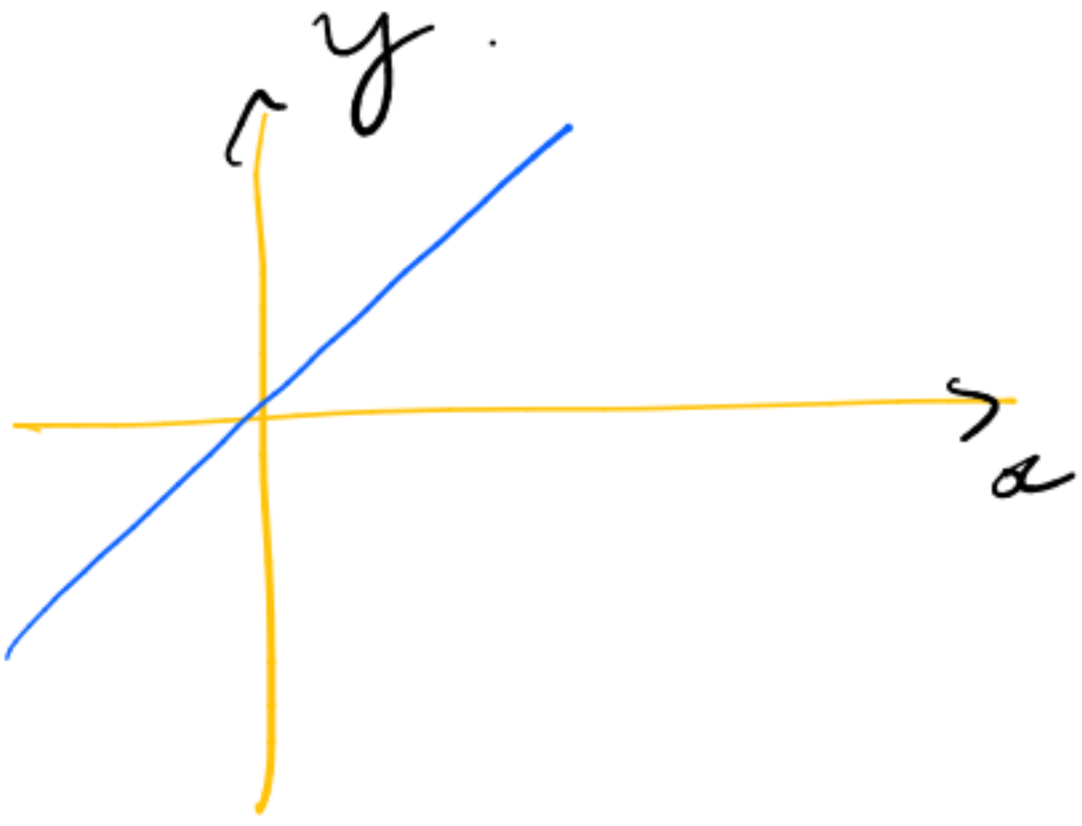
Plane polarised light:

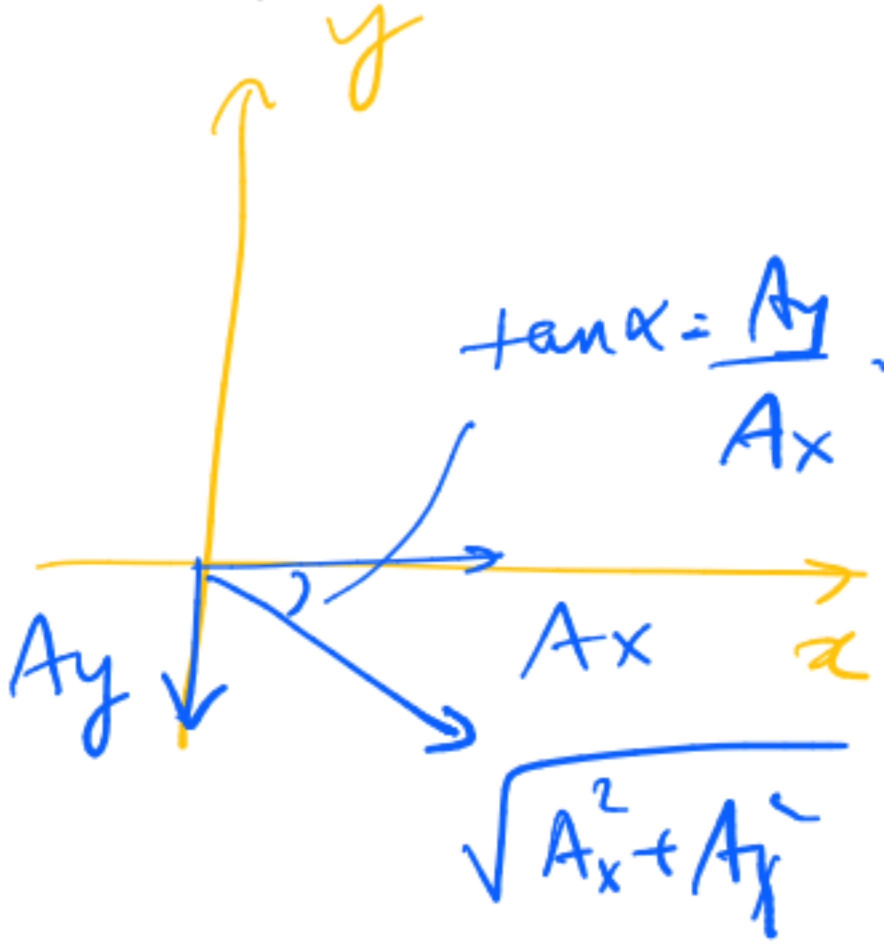
$\Delta = 0$ or π.

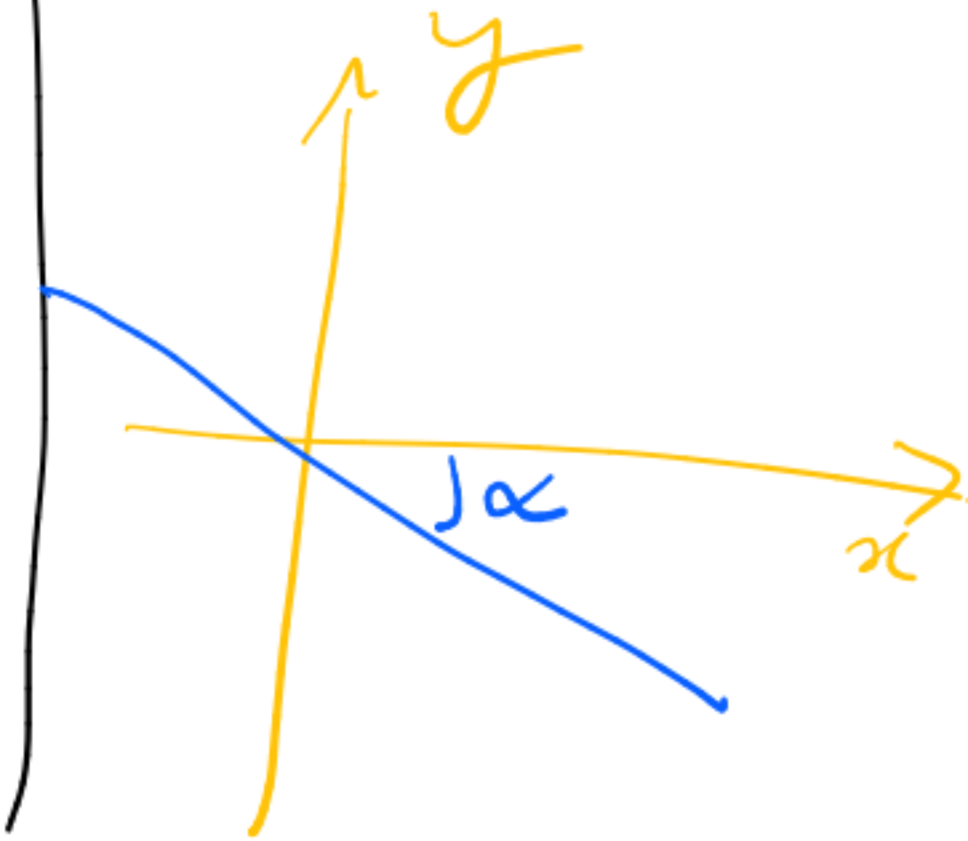
↙ ↘

$$\begin{pmatrix} A_x \\ A_y \end{pmatrix} \qquad \begin{pmatrix} A_x \\ -A_y \end{pmatrix}$$









AmazonBasics Circular Polarizer Filter- 55 mm

Brand: AmazonBasics

★★★★☆ 1,311 ratings

| 98 answered questions

M.R.P.: ~~₹ 1,045.00~~

Price: ₹ 539.00 ✓prime

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Circularly polarized light.

$$A_x = A_y$$

$$\Delta = \pm \pi/2$$

Light vector corresponding to circularly pol. light:

$$\begin{pmatrix} A_x \\ A_x e^{\pm i\frac{\pi}{2}} \end{pmatrix} = A_x \begin{pmatrix} 1 \\ \pm i \end{pmatrix}.$$

$$E_x = \mathrm{Re}\left[A_x \, e^{i\left(\frac{2\pi}{\lambda}(r-ct)\right)} \right]$$

$$E_x = A_x \cos\left(\frac{2\pi}{\lambda}(r-ct)\right).$$

$$E_y = \mathrm{Re}\left[\underset{\substack{\uparrow \\ \pm i A_x}}{A_y} \, e^{i\left(\frac{2\pi}{\lambda}(r-ct)\right)} \right]$$

$$= \mathrm{Re}\left[A_x(\pm i)\left(\cos\left(\frac{2\pi}{\lambda}(r-ct)\right) + i \sin\left(\frac{2\pi}{\lambda}(r-ct)\right)\right)\right]$$

$$E_y = \mp A_x \sin\left(\frac{2\pi}{\lambda}(r-ct)\right).$$

The E_x & E_y above satisfy

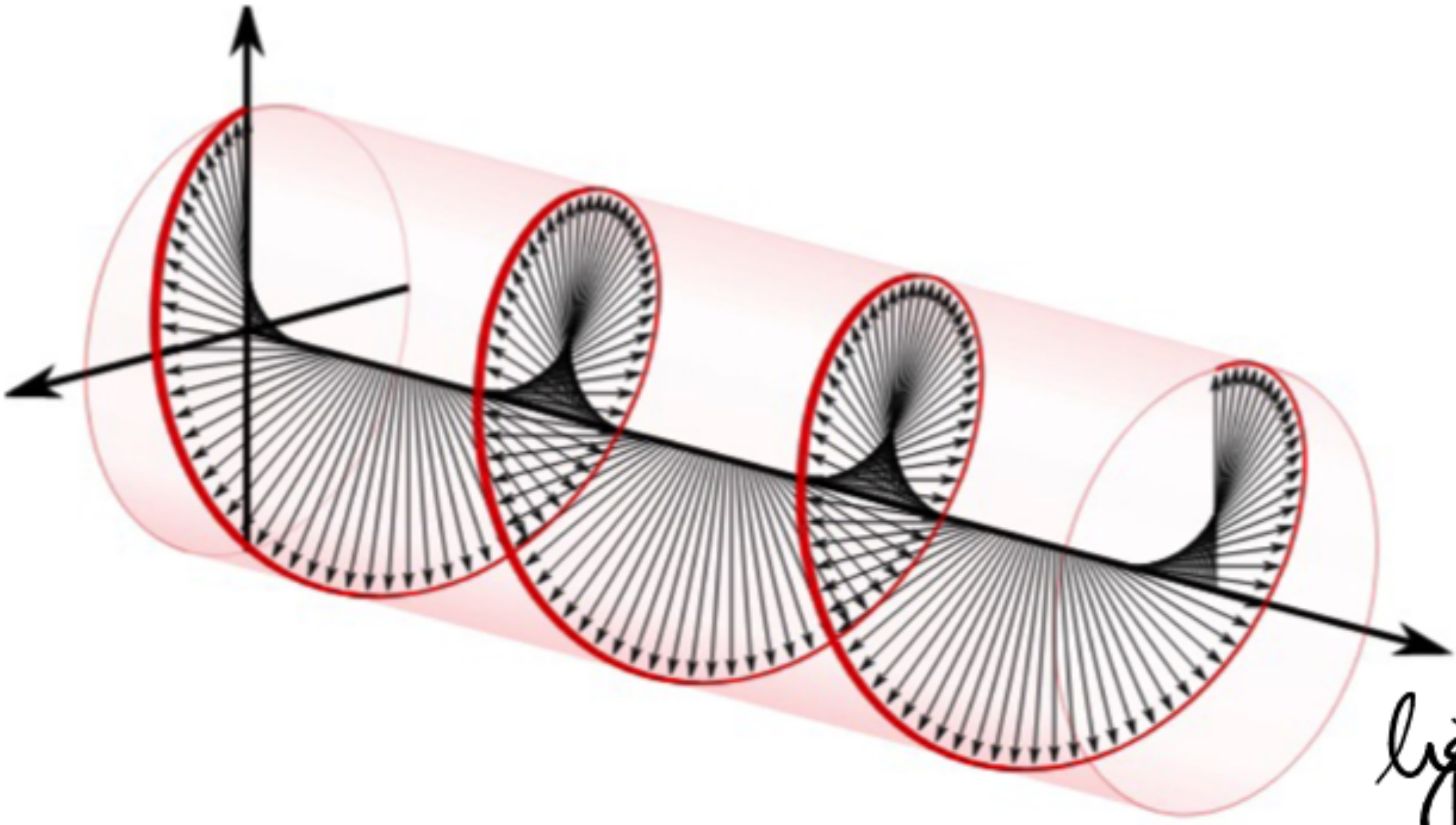
$$E_x^2 + E_y^2 = A_x^2.$$

Angle that the resultant vector makes with the x-axis

$$\tan\alpha = \frac{E_y}{E_x} = \mp \tan\left(\frac{2\pi}{\lambda}(r-ct)\right).$$

$$\Rightarrow \quad \alpha = \mp \frac{2\pi}{\lambda}(r-ct)$$

The angle the resultant E of constant magnitude A_x makes w/ the x-axis changes linearly with time.



light propagation direction.

Elliptically polarised light.

Light vector amplitude components along x- & y- axes.

$$\begin{pmatrix} A_x \\ \pm i A_y \end{pmatrix}$$

$$E_x = A_x \cos\left(\frac{2\pi}{\lambda}(r - ct)\right)$$

$$E_y = \cancel{\pm} A_y \sin\left(\frac{2\pi}{\lambda}(r - ct)\right)$$

$$\Rightarrow \frac{E_x^2}{A_x^2} + \frac{E_y^2}{A_y^2} = 1.$$

Phase angle depends also on E_y / E_x.

Plane polarizer.

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P = polarising axis of the polarising sheet

Incident light wave.

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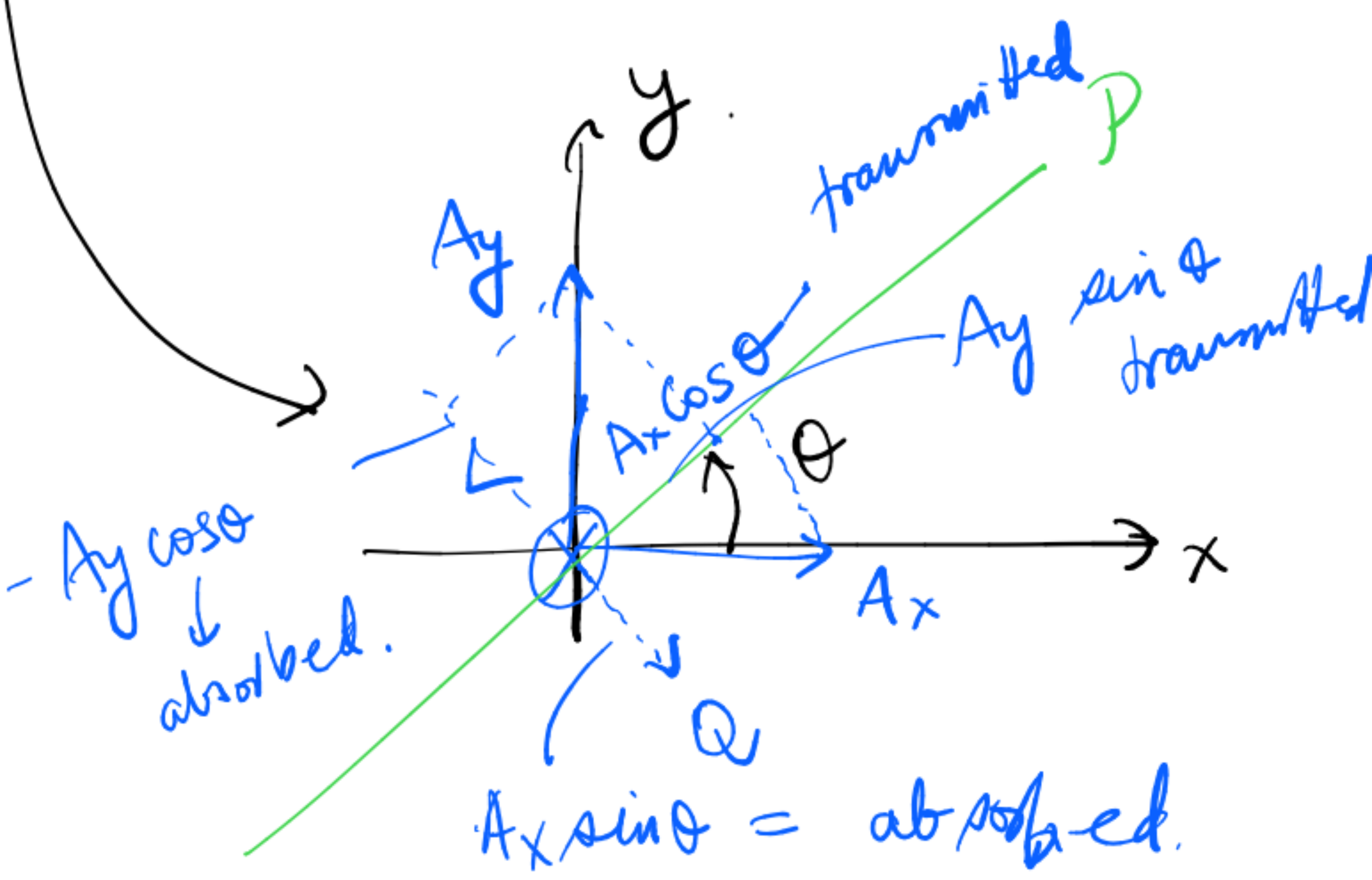


Emerging light will have electric field amplitude

$$\overset{\text{emerging}}{A'_p} = A_x \cos\theta + \overset{\text{incident}}{A_y} \sin\theta$$

along the polarizer axis. P

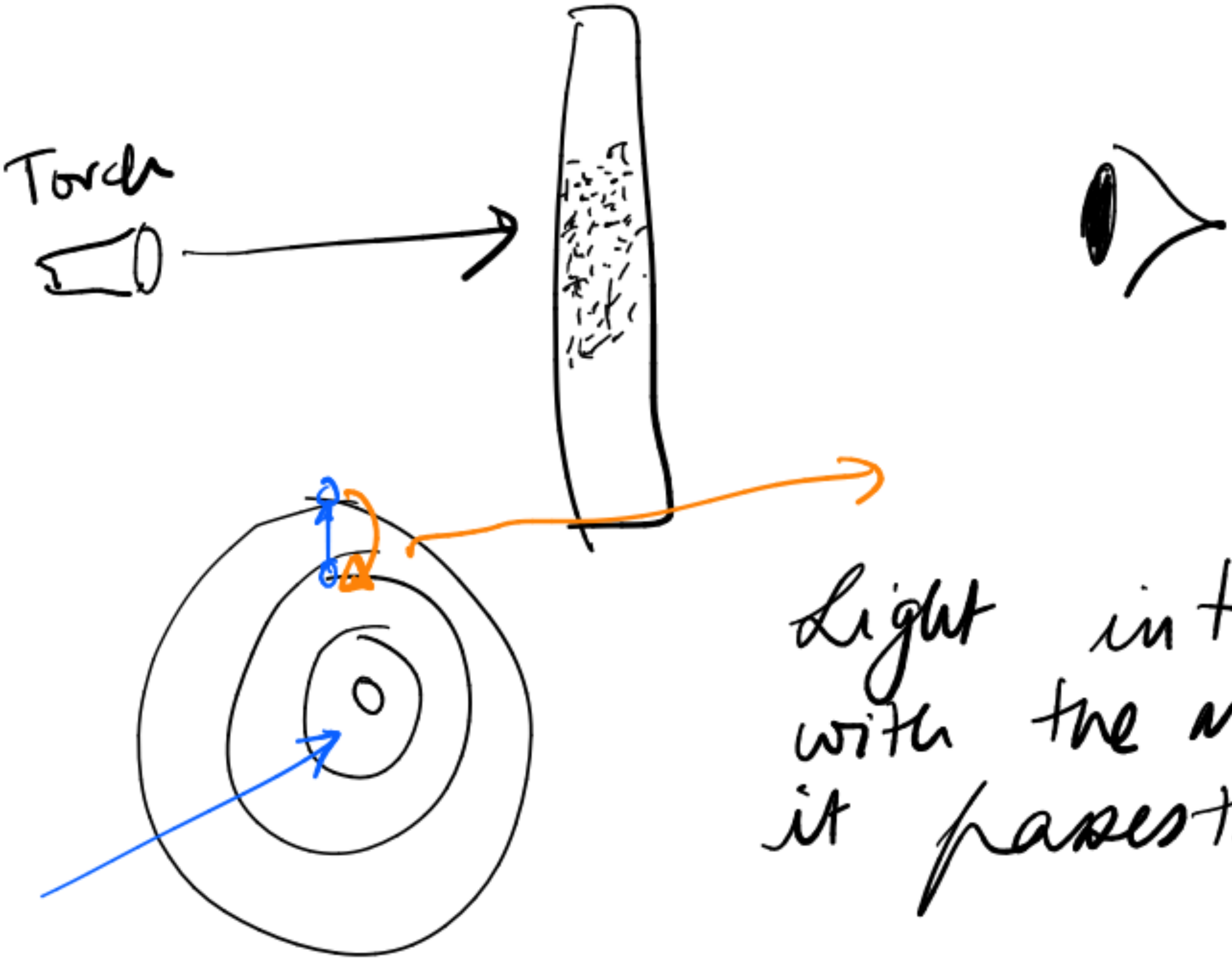
Components of the emerging light vector along the x- & y- axes = $\begin{pmatrix} A'_p \cos\theta \\ A'_p \sin\theta \end{pmatrix}$.

$$\Rightarrow \begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}}_{P_\theta \,=\, \text{polarizer matrix.}} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Common polarizer orientation is w/ $\theta = \pi/4$.

$$P_\theta = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Light in media other than vacuum.



Light interacts with the media it passes through.

Light travels slowly in non-vacuum media.

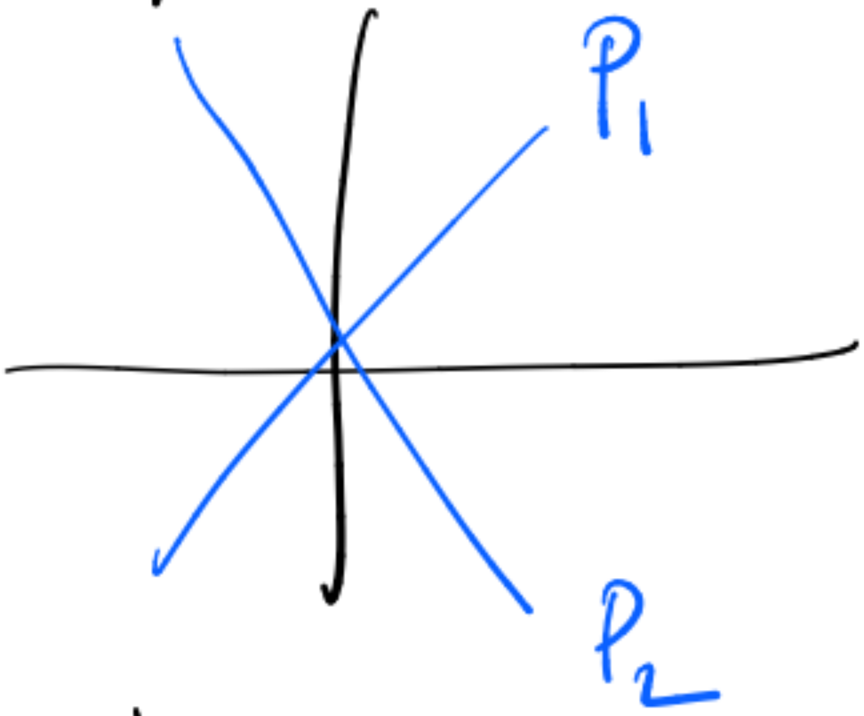
Speed of light in a medium $= \frac{c}{n}$,

where n = refractive index of the medium.

$n = 1.3$ for water
$n = 1.5$ for glass.

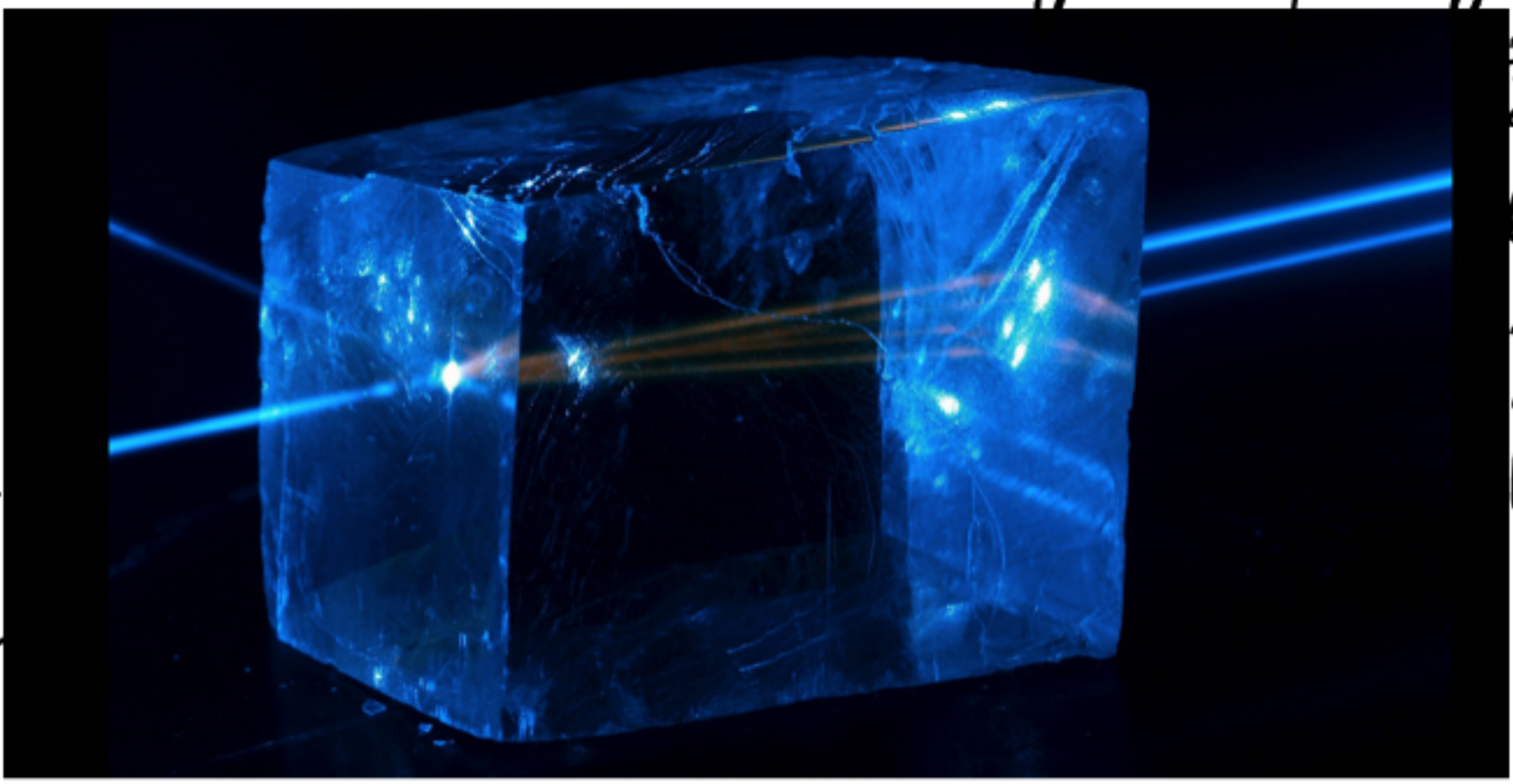
Birefringence.

Water is an isotropic transmitter of light ⇒ The speed of light in water does not depend on the polarization of light.



Calcite crystal: Speed of light / refracti index depends also on light polarization.

Incident light



Emergen light
↓
2 beams

The incident beam is split into multiple(2) emergent beams

— Light travels through calcite with difft. speeds depending on polarization.

Many non-crystalline transparent materials are optically isotropic (like water) when unloaded.

When loaded, however, they become anisotropic along the principal axes σ_1, σ_2, σ_3 of the stress tensor.

This phenomenon is called temporary or artificial birefringence.

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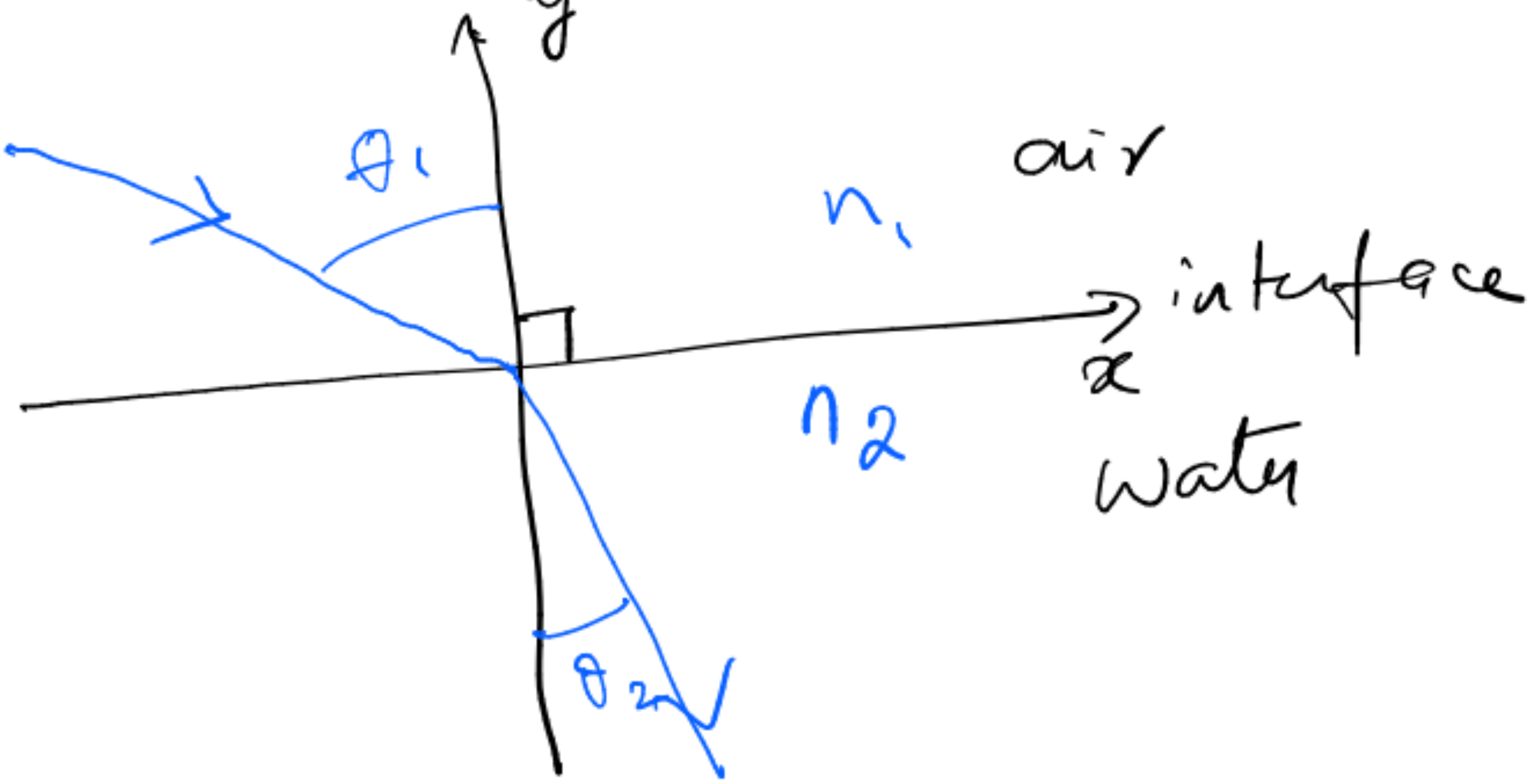
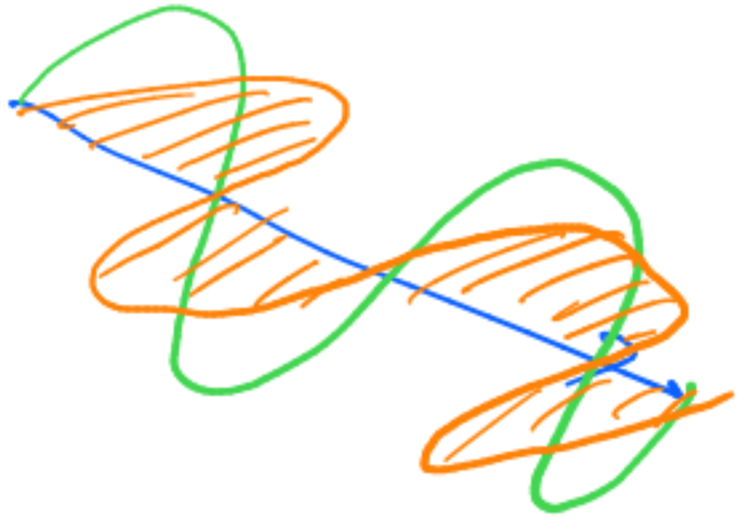
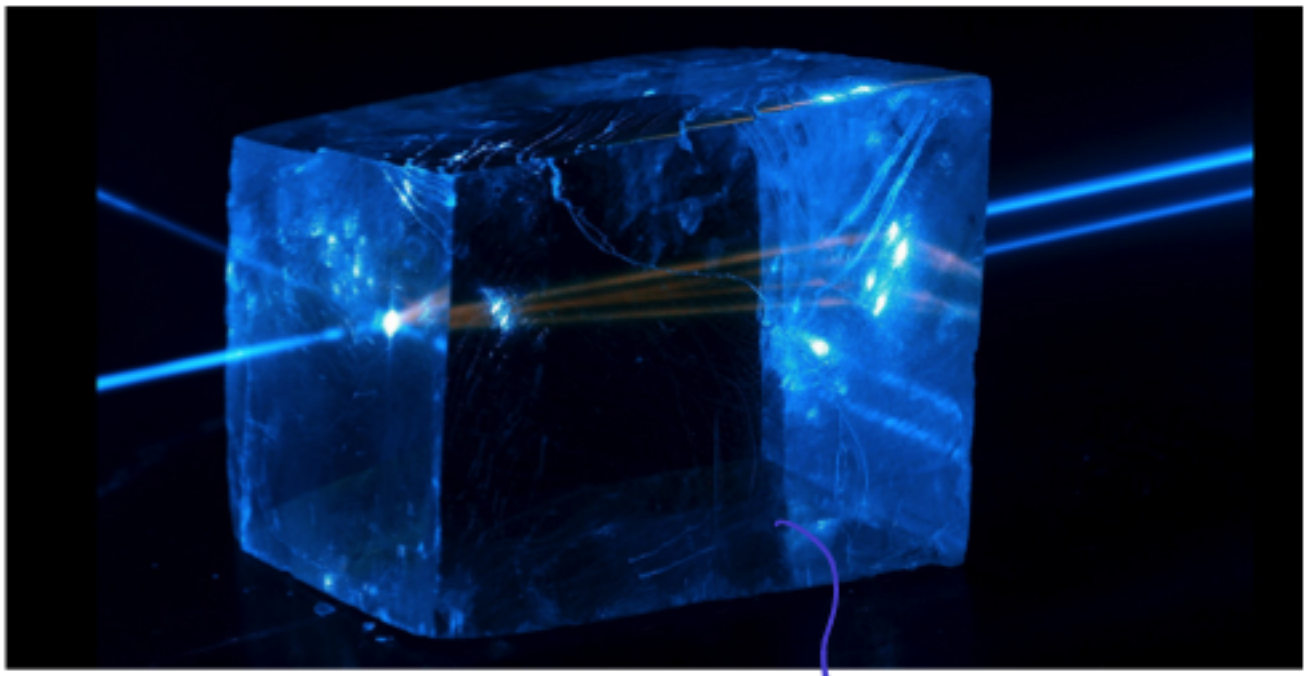
Lecture 8

Snell's law of refraction.



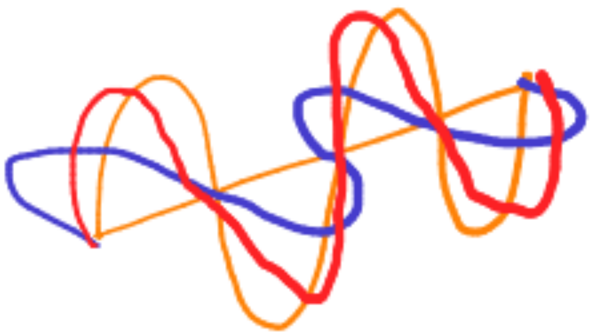
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$





Incident laser light has many polarities

optically anisotropic.



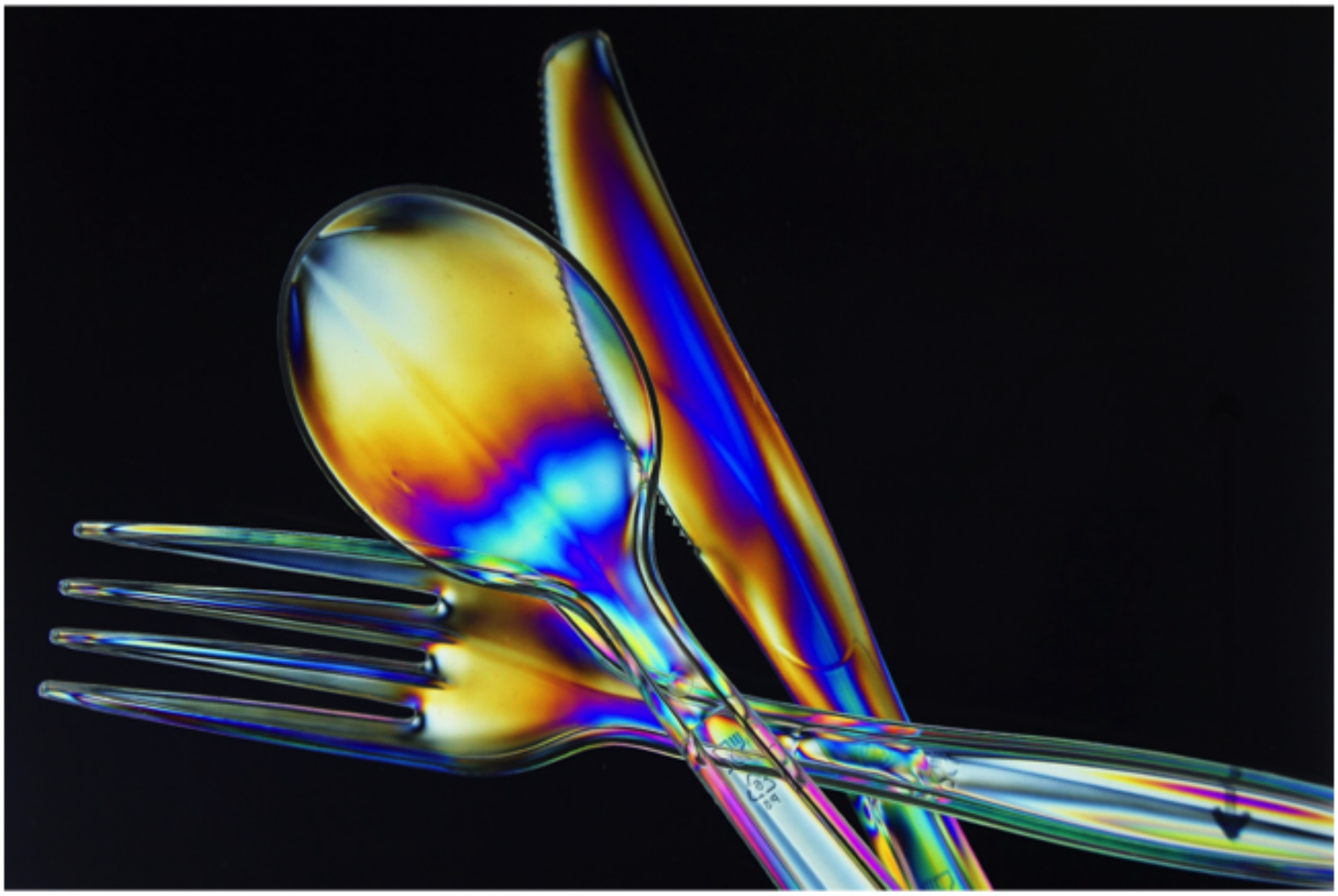
The calcite crystal is optically anisotropic.

This comes from the anisotropy of its crystal lattice.

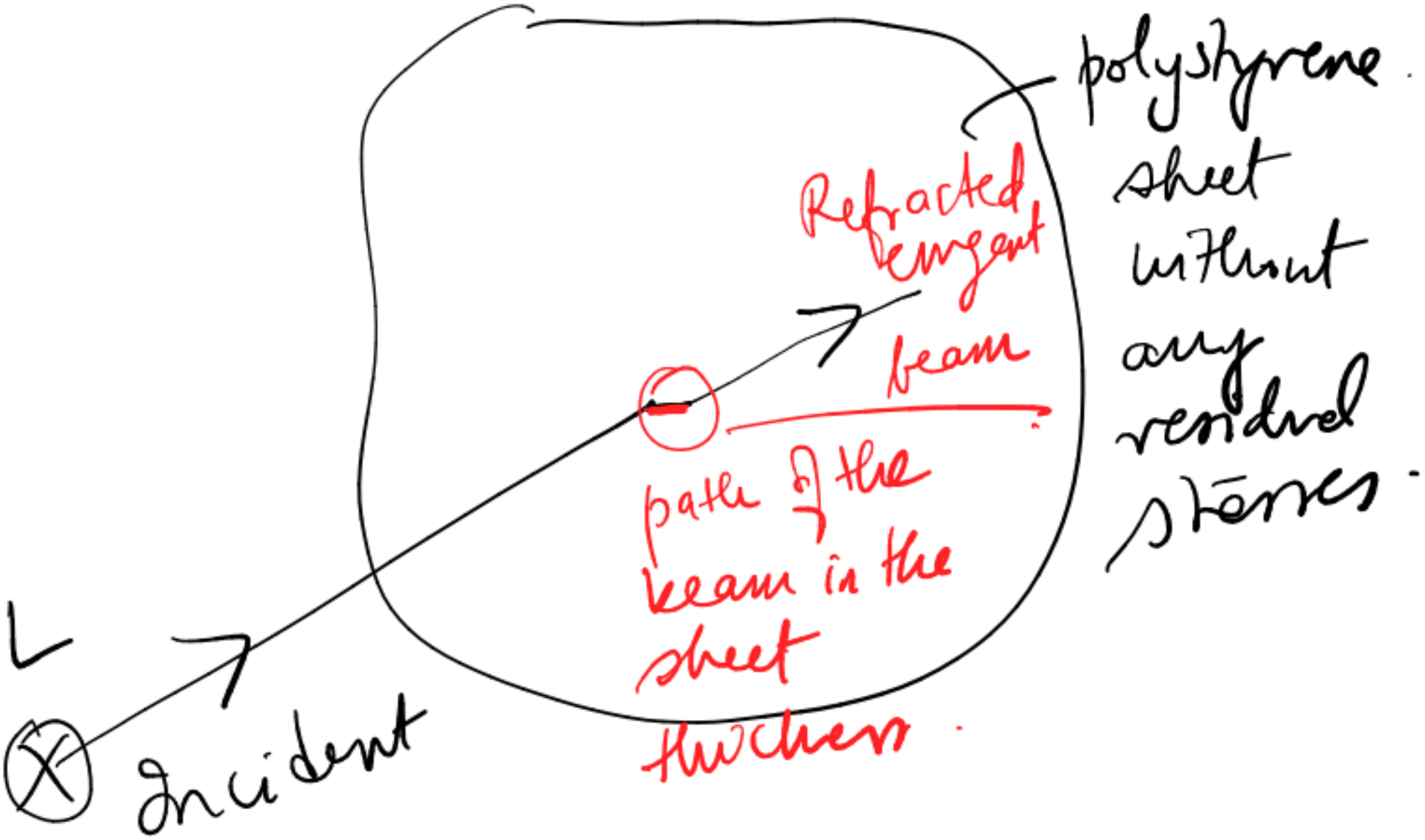
The calcite crystal displays a different refractive index for different polarities of the incident light.

The 2 beams travel along difft paths (according to Snell's law). and at diffent speeds.

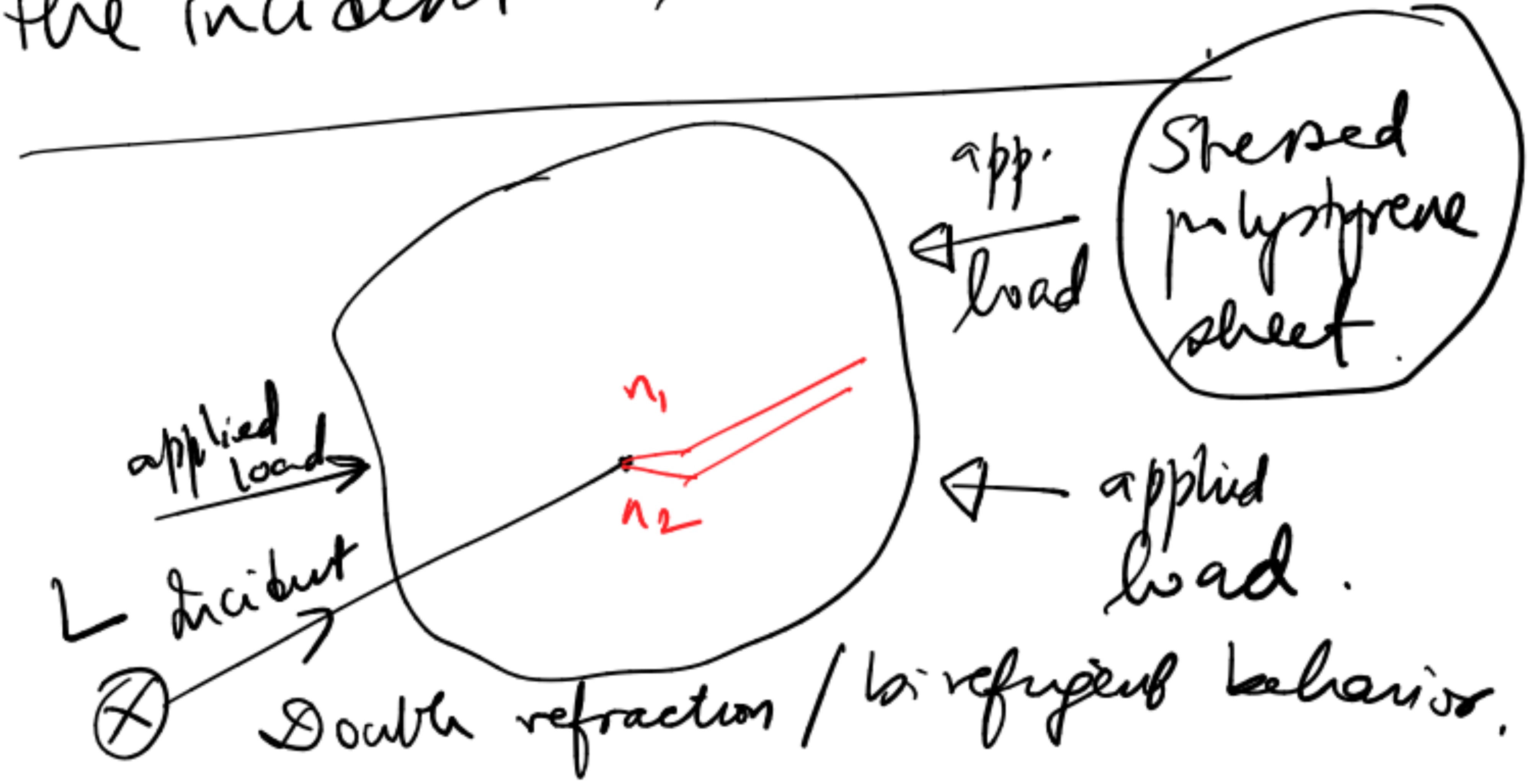
The property of calcite to display 2 refractive indices depending on the polarity of the incoming light beam is called double refraction or birefringence.



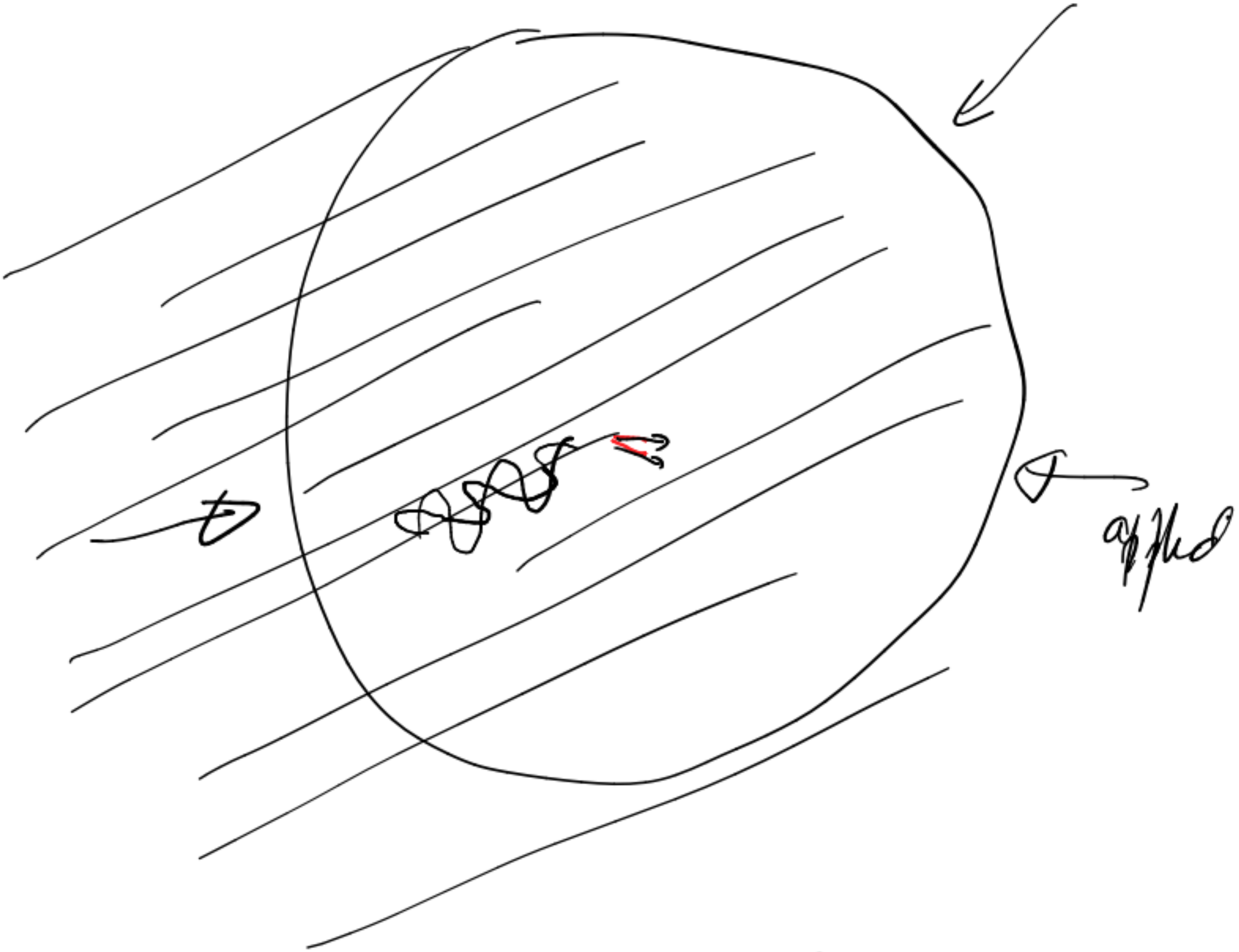
Plastics are also birefrigent because of their optical anisotropy which arises during their manufacture.



Single refraction — No splitting of the incident beam.



Double refraction / birefringent behavior.



If the entire face of the loaded plastic solid is illumied with white / monochromatic light, we will observe fringes (color / monochromatic) on the emergent side of the plastic birefringent plate.

Mathematically, a birefringent material introduces a phase shift, Δ between the 2 refracted waves.

Light beam passing through a birefringent sheet:

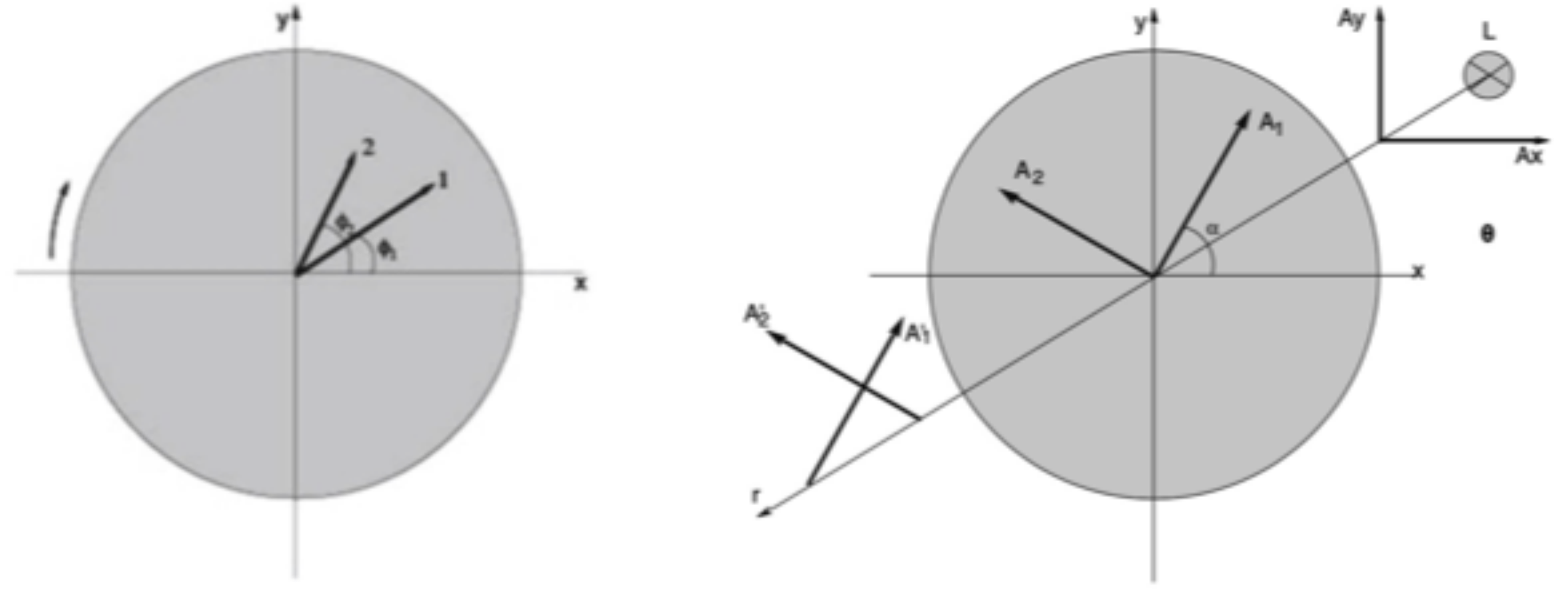


Fig. 3.6 Phase shift introduced by a birefringent filter in complex representation on the *left* and in physical space on the *right* ($\Delta = \Phi_2 - \Phi_1$)

Q: Given $\begin{pmatrix}A_x\\A_y\end{pmatrix}$ are the components of the incident light beam, find $\begin{pmatrix}A'_x\\A'_y\end{pmatrix}$ emerging from a birefringent sheet.

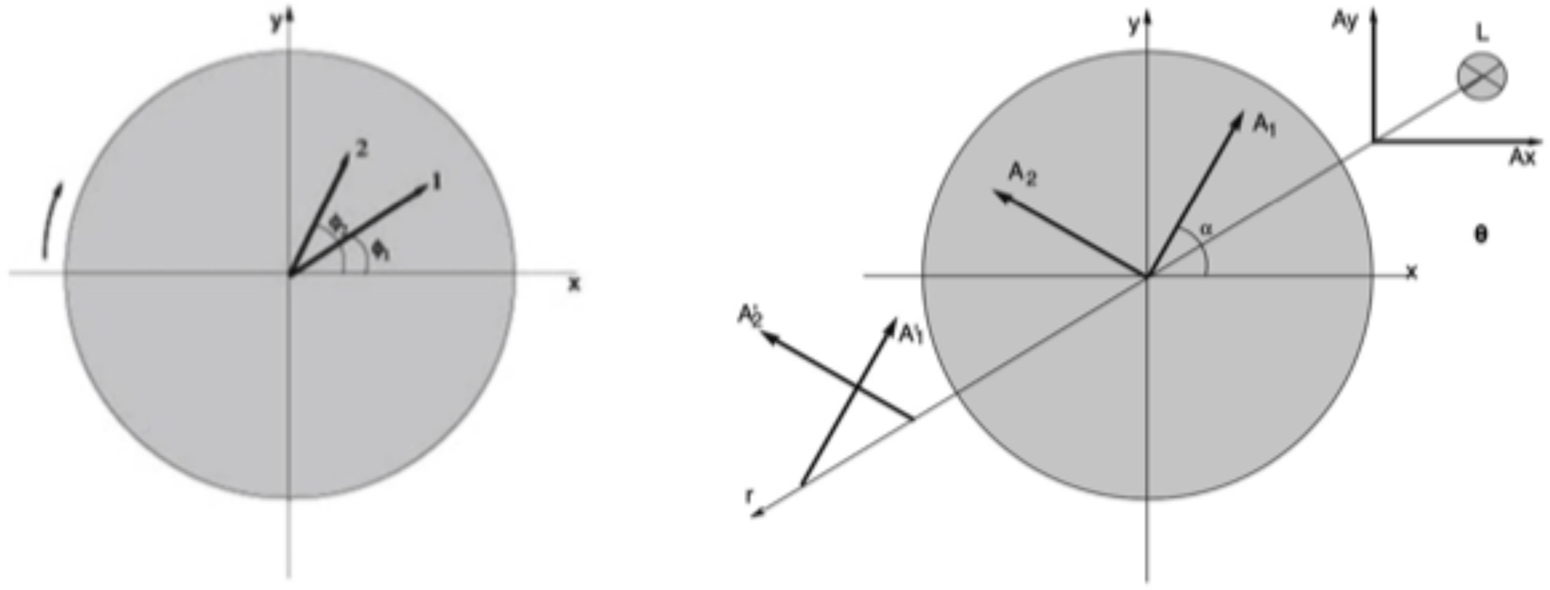
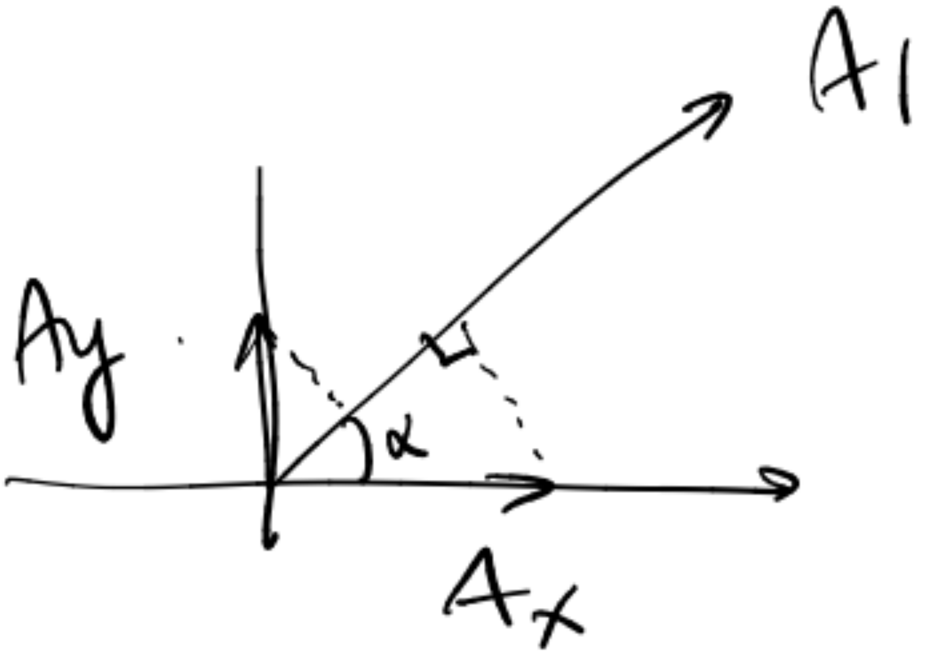


Fig. 3.6 Phase shift introduced by a birefringent filter in complex representation on the *left* and in physical space on the *right* ($\Delta = \Phi_2 - \Phi_1$)



$$A_1 = A_x \cos\alpha + A_y \sin\alpha.$$
$$A_2 = -A_x \sin\alpha + A_y \cos\alpha.$$

A_1, A_2 are complex components along 1,2 axes of the incident beam.

Unlike a polarizing sheet, a birefringent sheet introduces a phase shift between the A_1 + A_2 waves.

← emergent

$$\left.\begin{aligned} A_1' &= A_1 \\ A_2' &= A_2 \underbrace{e^{i\Delta}}_{\text{phase shift}} \end{aligned}\right\} \text{①}$$

introduced by difft. lights speeds for polarity along A_1 & A_2.

$$\begin{pmatrix} A_1' \\ A_2' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

②

① & ② are 2 forms of the same equation.

Rotate A_1', A_2' back to x y coord. system:

$$\underset{\substack{\uparrow \\ \text{transmitted} \\ \text{beam}}}{\begin{pmatrix} A'_x \\ A'_y \end{pmatrix}} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\Delta} \end{pmatrix} \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \underset{\substack{\uparrow \\ \text{incident} \\ \text{beam}}}{\begin{pmatrix} A_x \\ A_y \end{pmatrix}}.$$

More Compactly:

$$\boxed{A' = M_{2\times 2} A}$$

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} \qquad \qquad \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

$$\left(\begin{array}{c|c} \cos^2\alpha + e^{i\Delta}\sin^2\alpha & (1-e^{i\Delta})\sin\alpha\cos\alpha \\ \hline (1-e^{i\Delta})\sin\alpha\cos\alpha & \sin^2\alpha + e^{i\Delta}\cos^2\alpha \end{array}\right)$$

If the birefringent sheet produces

$\Delta = \frac{\pi}{2}$, it is called a quarter wave plate.

$\Delta = \pi$, half wave plate.

$\Delta = 2\pi$, full wave plate.

M for a quarter wave plate:

$$Q = \underset{\substack{\uparrow \\ \Delta = \pi/2 \\ \alpha = \pm\pi/4}}{M} = \frac{1}{2}\begin{pmatrix} (1+i) & \pm(1-i) \\ \pm(1-i) & (1+i) \end{pmatrix}.$$

$$1+i = 2e^{i\pi/4}.$$

$$1-i = 2e^{-i\pi/4}$$

$$= \frac{1}{\cancel{2}}\begin{pmatrix} \cancel{2}e^{i\pi/4} & \pm\cancel{2}e^{-\frac{i\pi}{4}} \\ \pm\cancel{2}e^{-i\pi/4} & \cancel{2}e^{i\pi/4} \end{pmatrix}$$

$$= \boxed{e^{i\pi/4}}\begin{pmatrix} 1 & \pm e^{-i\pi/2} \\ \pm e^{-\frac{i\pi}{2}} & 1 \end{pmatrix}$$

$$\boxed{\text{neglect}} \leftarrow \boxed{e^{i\pi/4}}$$

$$= \boxed{e^{i\pi/4}}\begin{pmatrix} 1 & \mp i \\ \mp i & 1 \end{pmatrix}.$$

$\begin{pmatrix} 1 & \mp i \\ \mp i & 1 \end{pmatrix}$ is the quarter-wave plate transfer function.