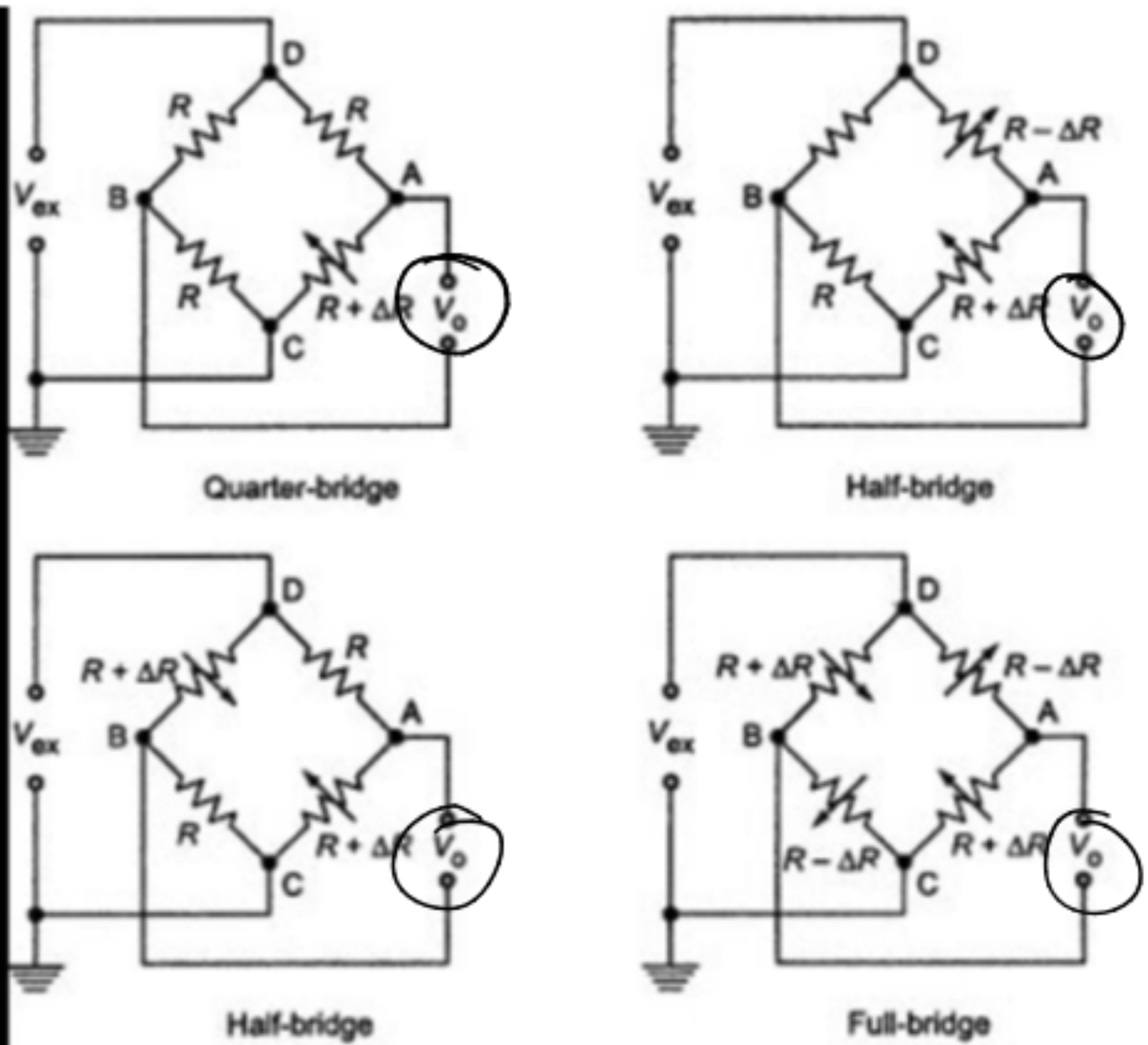
AS 3520

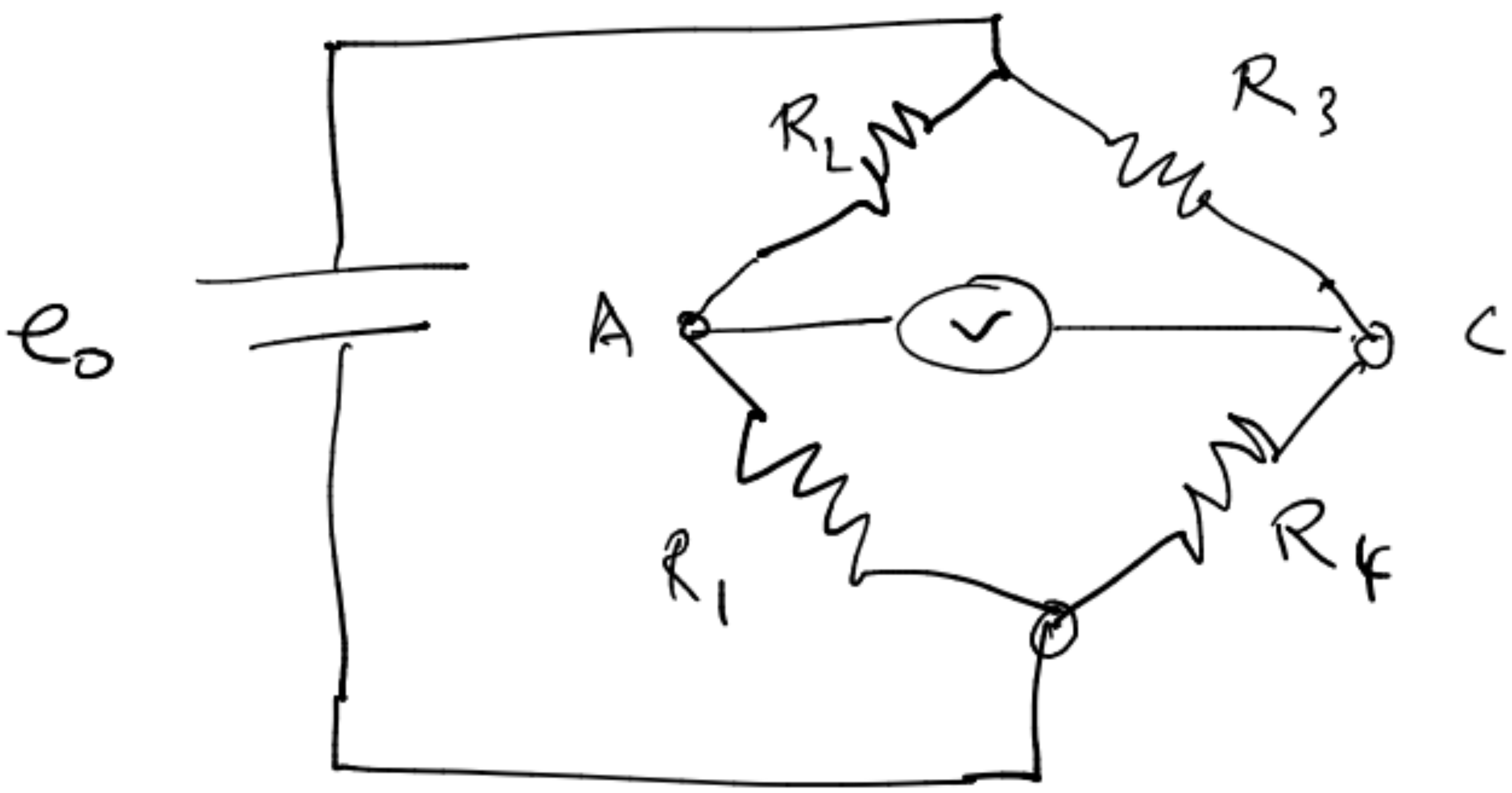
Lecture 5

Potentiometric circuits.

Wheatstone bridge



Q. Given a balanced bridge, how do changes in the resistances of the various elements affect the voltmeter loading.



We start with a balanced bridge so V reads zero at the beginning (prior to loading)

$$1 - \frac{R_3}{R_3+R_4} = 1 - \frac{R_2}{R_1+R_2} \quad \Bigg\| \quad \frac{R_4}{R_3+R_4} = \frac{R_1}{R_1+R_2}$$

$$\Rightarrow \quad R_3 R_1 + \cancel{R_3 R_2} = \cancel{R_3 R_2} + R_2 R_4$$

$$\Rightarrow \quad R_1 R_3 = R_2 R_4.$$

Let the resistances change

$$R_1 \longrightarrow R_1 + \Delta R_1$$
$$R_2 \longrightarrow R_2 + \Delta R_2$$
$$R_3 \longrightarrow R_3 + \Delta R_3$$
$$R_4 \longrightarrow R_4 + \Delta R_4$$
$$e_0 \longrightarrow e_0 + \Delta e_0$$

Let the V reading be e.

In the balanced bridge $e = 0$.

We want a formula for Δe due to $\Delta e_0, \Delta R_1, \dots, \Delta R_4$.

$$e = V_C - V_A = \left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) e_0$$

$$de = \frac{\partial e}{\partial R_1} dR_1 + \cdots + \frac{\partial e}{\partial R_4} dR_4 + \frac{\partial e}{\partial e_0} de_0.$$

$$\frac{\partial e}{\partial R_1} = -e_0 \frac{\partial}{\partial R_1}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$= -e_0 R_1 \frac{\partial}{\partial R_1}(R_1 + R_2)^{-1}$$

$$= \frac{e_0 R_1}{(R_1 + R_2)^2} \cdot \frac{\partial}{\partial R_1}(R_1 + R_2)$$

$$= \frac{e_0 R_1}{(R_1 + R_2)^2}$$

$$\frac{\partial e}{\partial R_1} dR_1 = \frac{e_0 R_1}{(R_1 + R_2)^2} dR_1$$

$\|^{ly}$, get $\frac{\partial e}{\partial R_2} dR_2$, etc..

$$\Delta e = \frac{R_1 R_2 e_0}{(R_1 + R_2)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right)$$

Volt meter reading

$$+ \frac{R_3 R_4 e_0}{(R_3 + R_4)^2}\left(\frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right)$$

$$+ \underbrace{\left(\frac{R_3}{R_3 + R_4} - \frac{R_2}{R_1 + R_2}\right)}_{0} \Delta e_0 .$$

$\because$ bridge was initially balanced.

$$\Rightarrow \frac{\Delta e}{e_0} = \frac{R_1 R_2}{(R_1 + R_2)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right) + \frac{R_3 R_4}{(R_3 + R_4)^2}\left(\frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right).$$

equal for a balanced bridge.

$$\Rightarrow \frac{\Delta e}{e_0} = \frac{R_1 R_2}{(R_1+R_2)^2}\left\{\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right\}$$

If $\left.\begin{matrix} R_1 = R_2 \\ R_3 = R_4 \end{matrix}\right\}$ Note : Not demanding equality of $\Delta R_1, \Delta R_2$ etc

$$\boxed{\frac{\Delta e}{e_0} = \frac{1}{4}\left\{\frac{\Delta R_1 - \Delta R_2}{R_1} + \frac{\Delta R_3 - \Delta R_4}{R_3}\right\}}$$

↑ circuit factor.

Quarter bridge

$R_1 \longrightarrow$ strain gauge.

$R_1 \longrightarrow R_1 + \Delta R_1$

$\Delta R_2 = \Delta R_3 = \Delta R_4 = 0$

$\boxed{\frac{\Delta R}{R} = S\varepsilon}$

$$\frac{\Delta e}{e_0} = \frac{1}{4}\frac{\Delta R_1}{R_1} = \frac{S\varepsilon}{4}.$$

measured by V

$$\frac{\Delta e}{e_0} = \frac{S\varepsilon}{4}$$

unknown

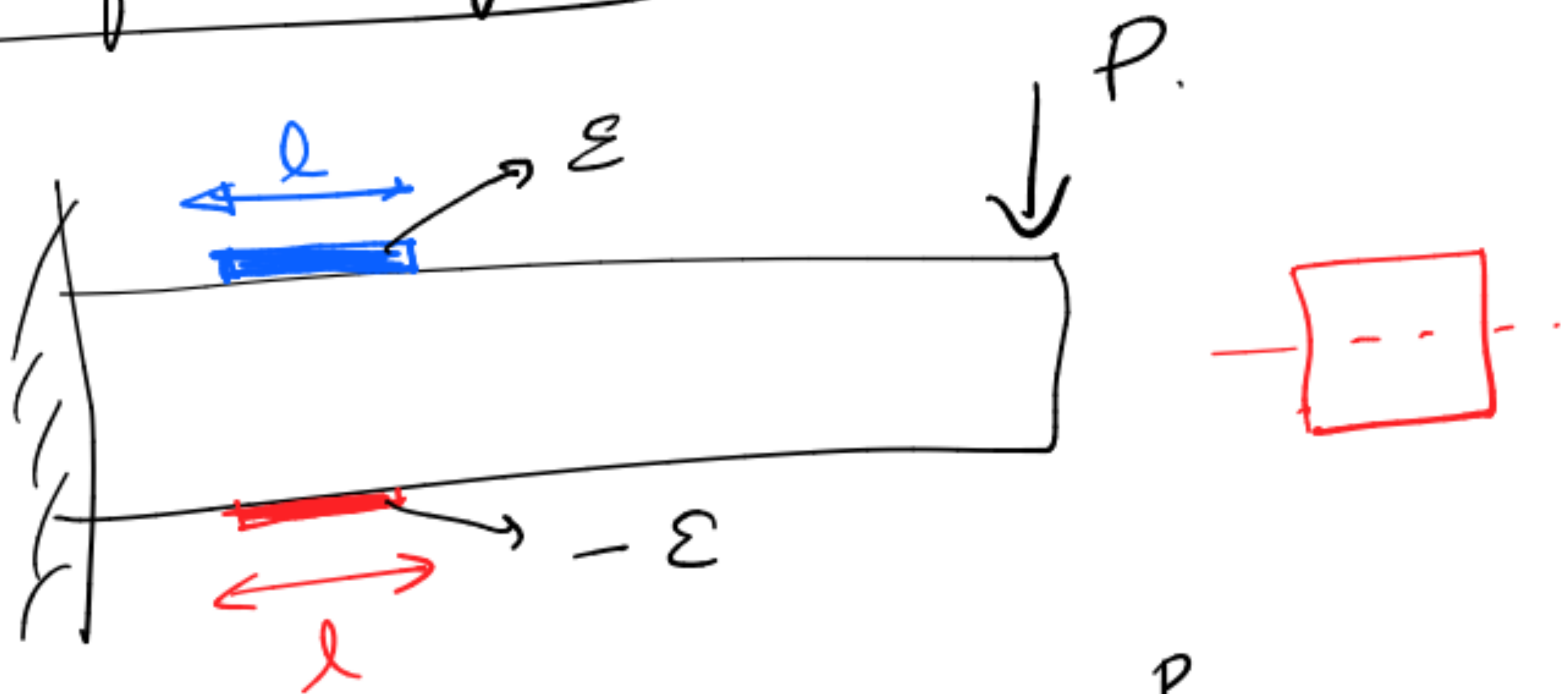
applied e.m.f.

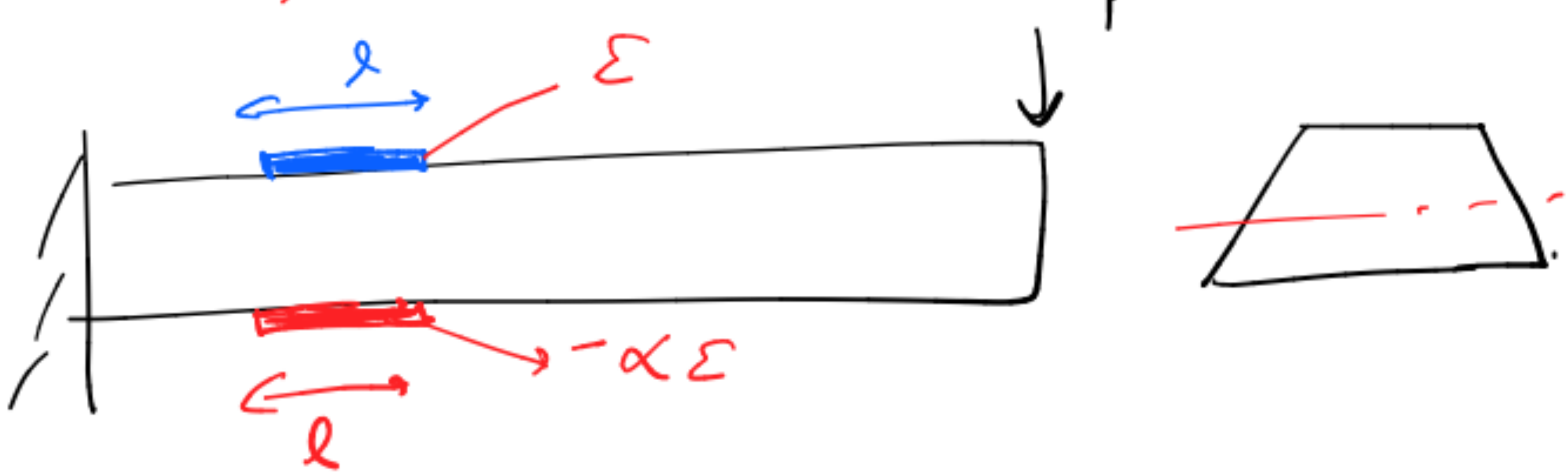
guage factor

$$\Rightarrow \boxed{\varepsilon = \frac{4}{S}\,\frac{\Delta e}{e_0}}$$

for a quarter bridge.

Half bridge :





Let us assume the 2 SGs pick up $\pm \varepsilon$ strain.

Use this information in formula ✪ of page 6.

$$\Delta R_1 = \Delta R_2 = 0.$$

$$\frac{\Delta R_3}{R_3} = S \varepsilon$$

upper guage

$$\frac{\Delta R_4}{R_4} = -S \varepsilon$$

lower guage

Coming from BSM theory.

reading ✓

unknown strain

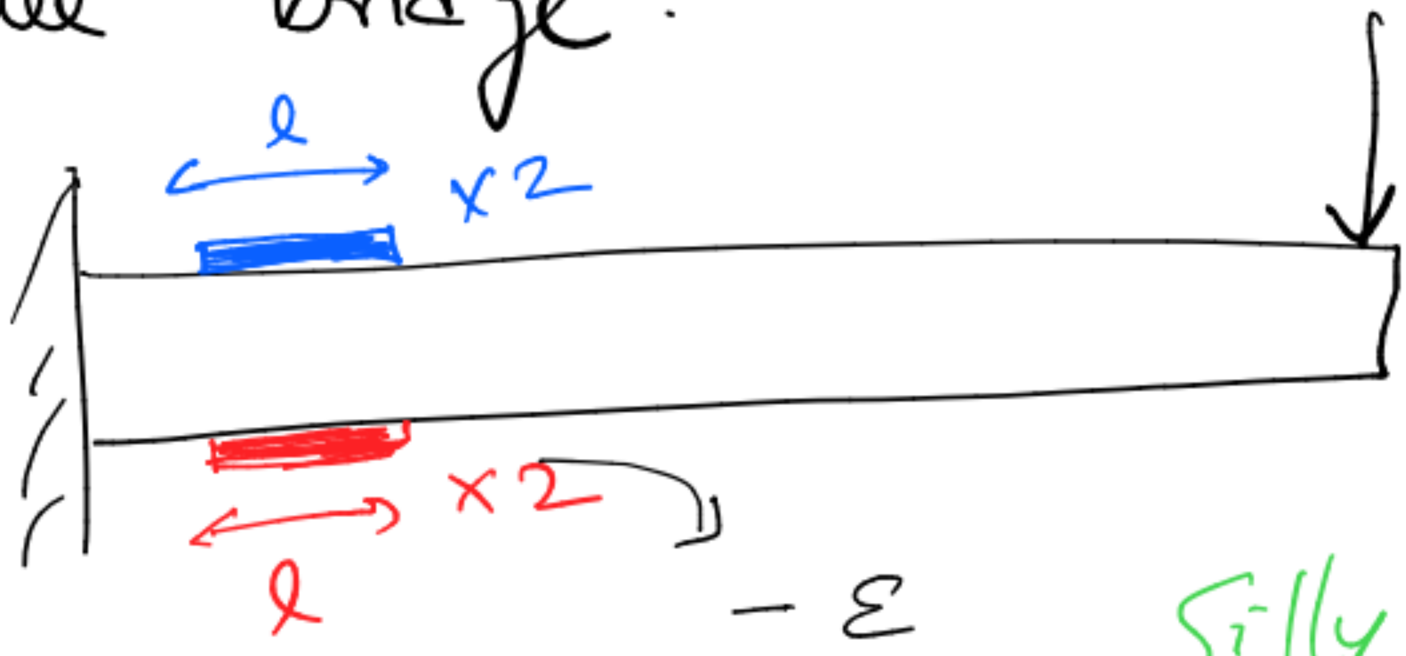
$$\frac{\Delta e}{e_0} = \frac{1}{4}\left\{ 2 S \varepsilon \right\} = \frac{1}{2} S \varepsilon.$$

applied emf

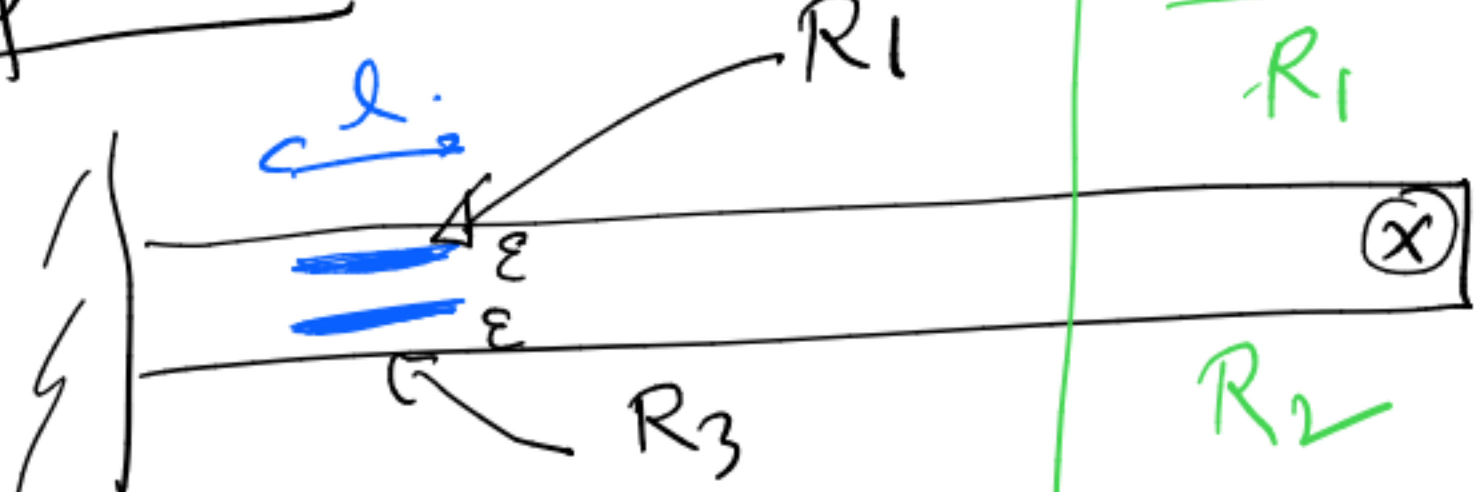
Sensitivity.

$$\boxed{\varepsilon = \frac{2}{S} \frac{\Delta e}{e_0}}$$

Full bridge.



Topview:



Silly way:

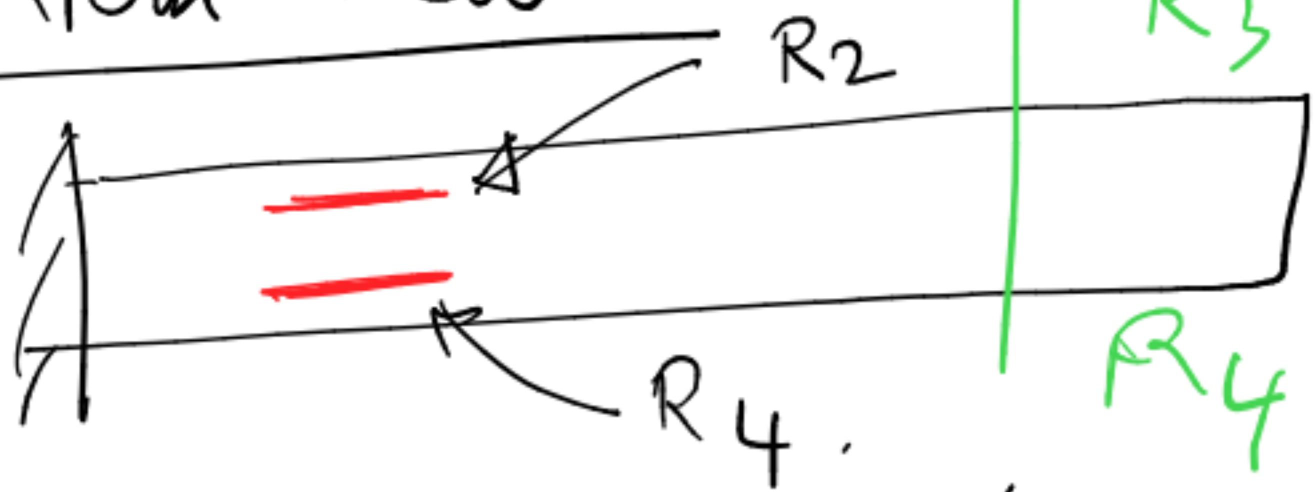
R1

R2

R3

R4

Bottom view:



Using Eq ✪ on page 6,

$$\frac{\Delta e}{e_0} = \frac{1}{\cancel{4}}\left\{ \cancel{4}\, S\, \varepsilon \right\}$$

$$\Rightarrow \boxed{\varepsilon = \frac{1}{S}\,\frac{e}{e_0}}$$ for a full bridge.

Load Cell:

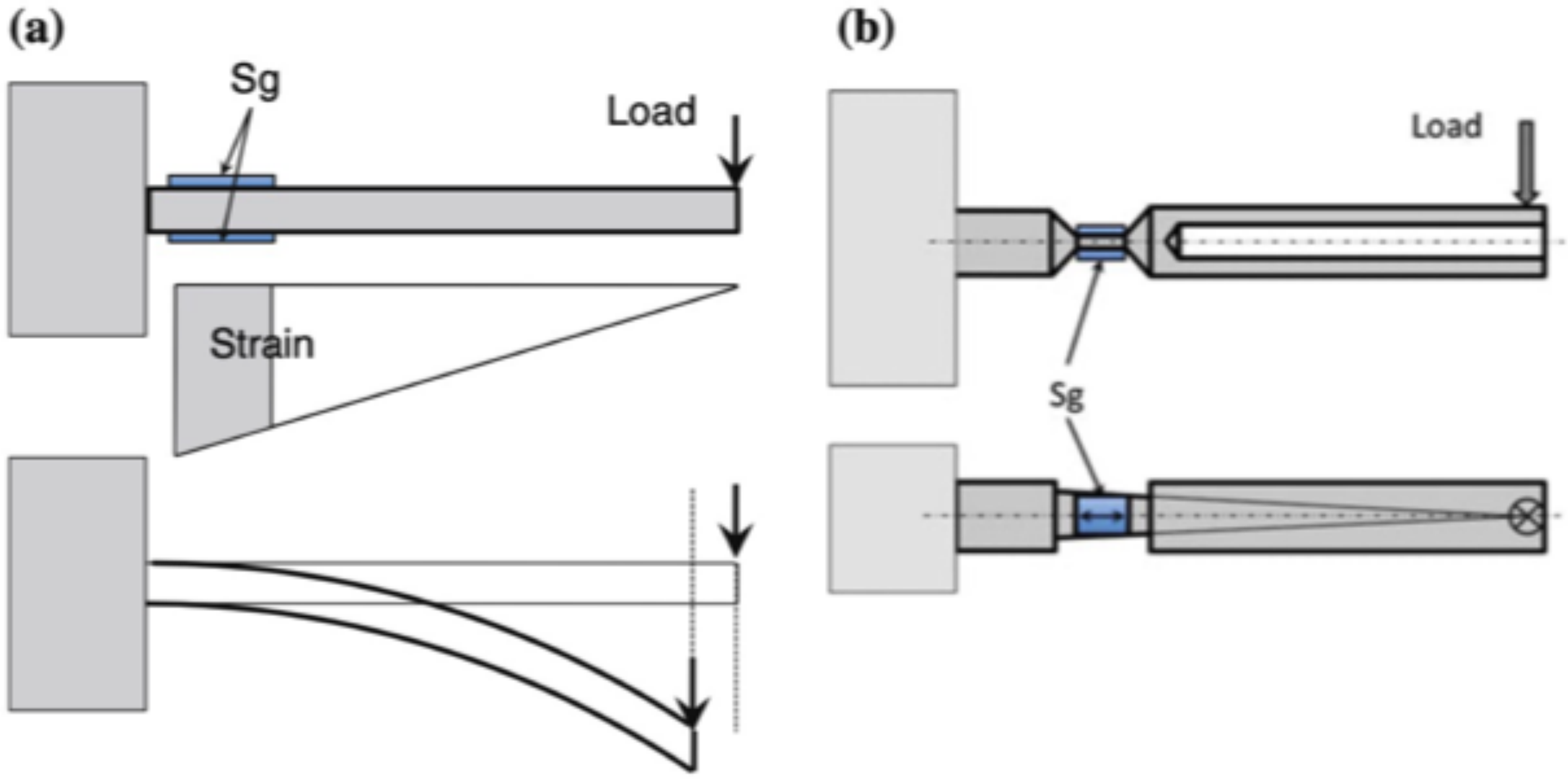
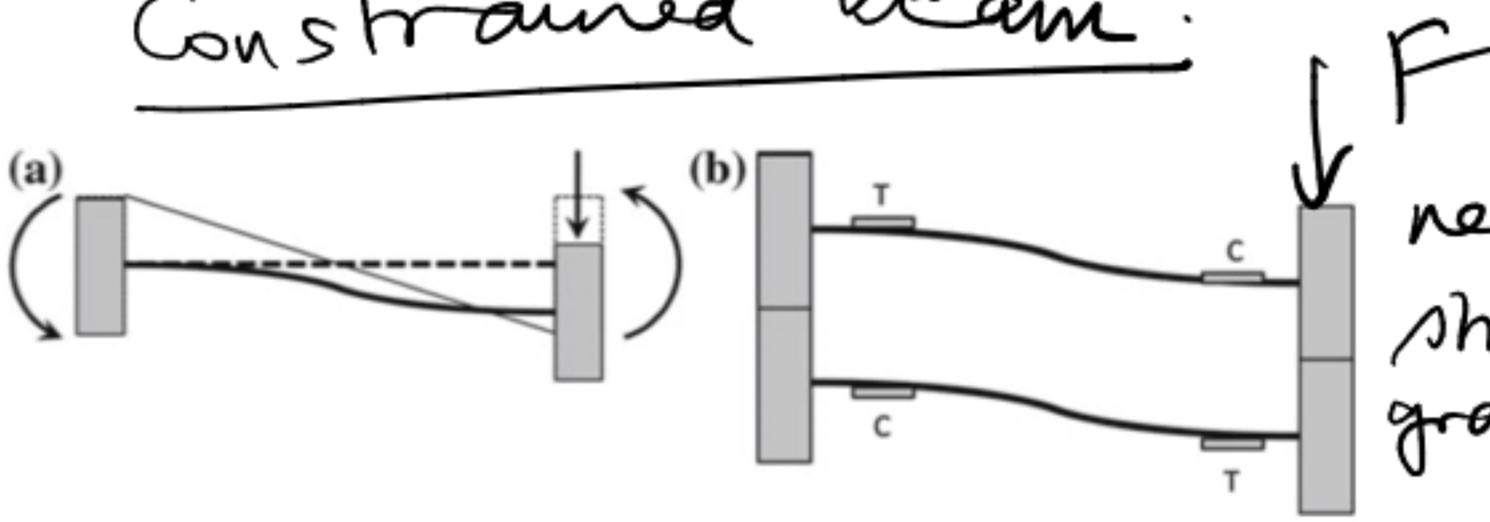


Fig. 2.27 The simplest example of the bending cell. **a** A cantilever beam with a load at one end, instrumented with two opposing strain gages and **b** an improved development of the basic idea by reducing the thickness and tapering the measuring area in order to make the strains in the gage grid uniform

Constrained beam:



negligible strain gradient.

Fig. 2.28 Static schemes of load cells with single beam: **a** doubly fixed, with loads and moments applied to the opposite ends and **b** with double beam, instrumented with four strain gages on the opposite sides