AS 5900: Elasticity

Lecture #1.

Sadd — 2 edition, 2009

Chap 1 – 9

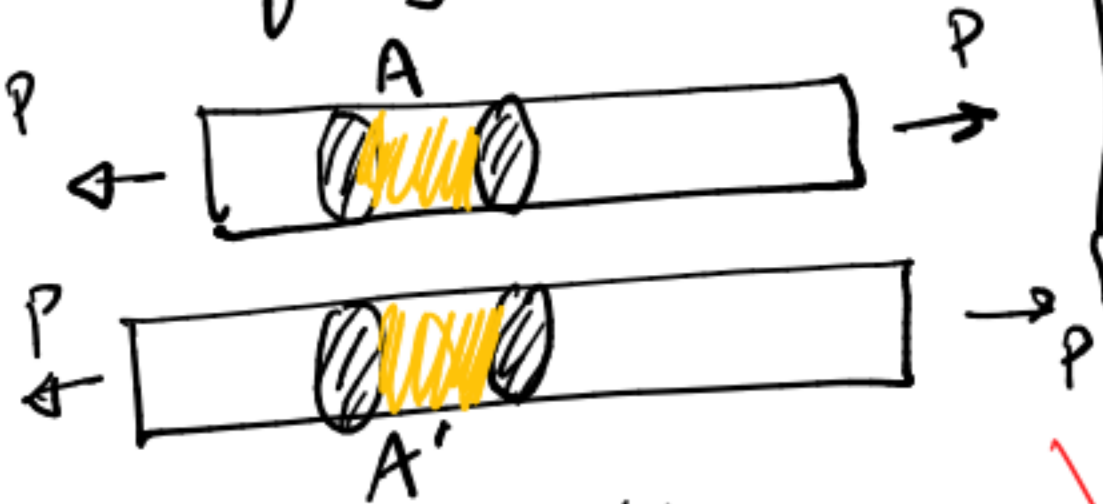
Attendance — don't take.

Exams — Dean Academic.

Strength of materials

2nd year BTech

Rods, Beams, Shafts } Common feature { Slender



A is a plane

A'' is also a plane. } Kinematic assumption

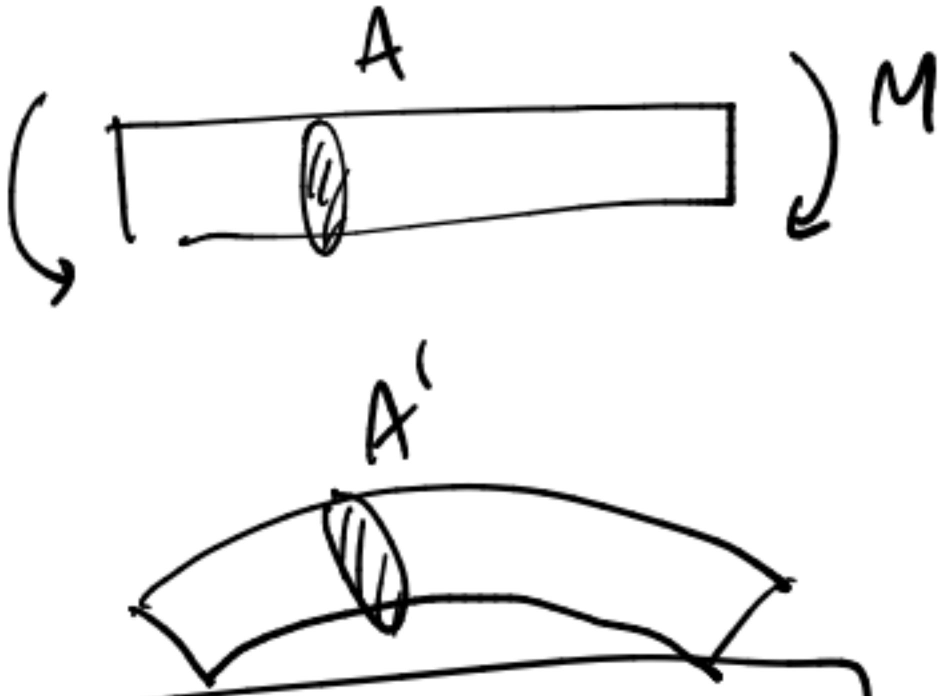
Elasticity.

Later

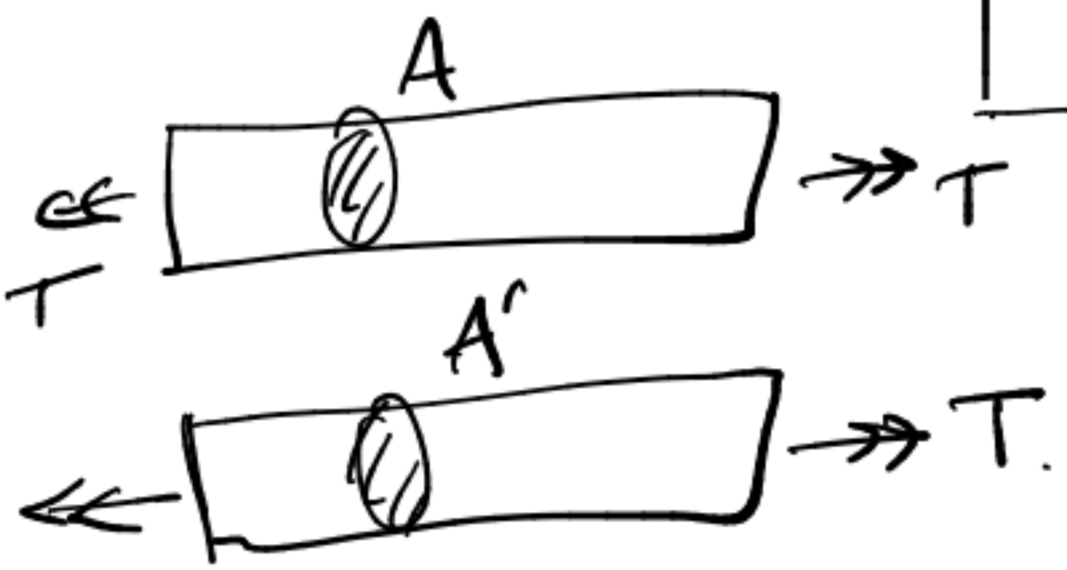
↳ more advanced.

We deal with ARBITRARY Objects (not necessarily slender).

Strength of M



A is a plane
A' is also plane



A, A' both planar
if A were circular

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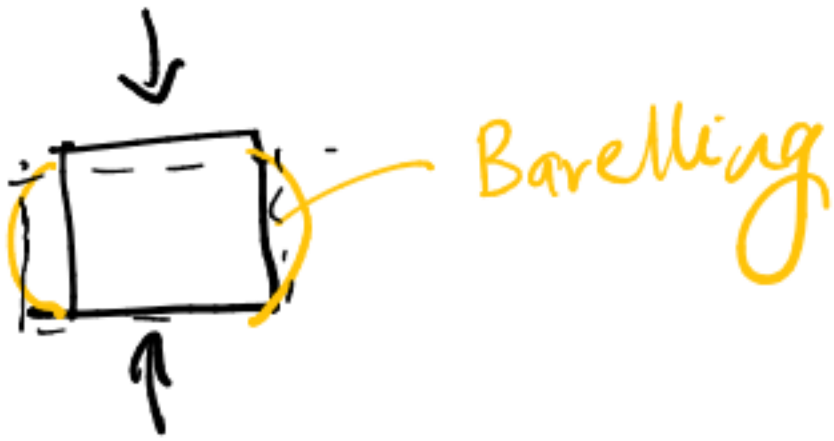
planarity assumption of A &
A' is OK only if
members are slender.

Elasticity

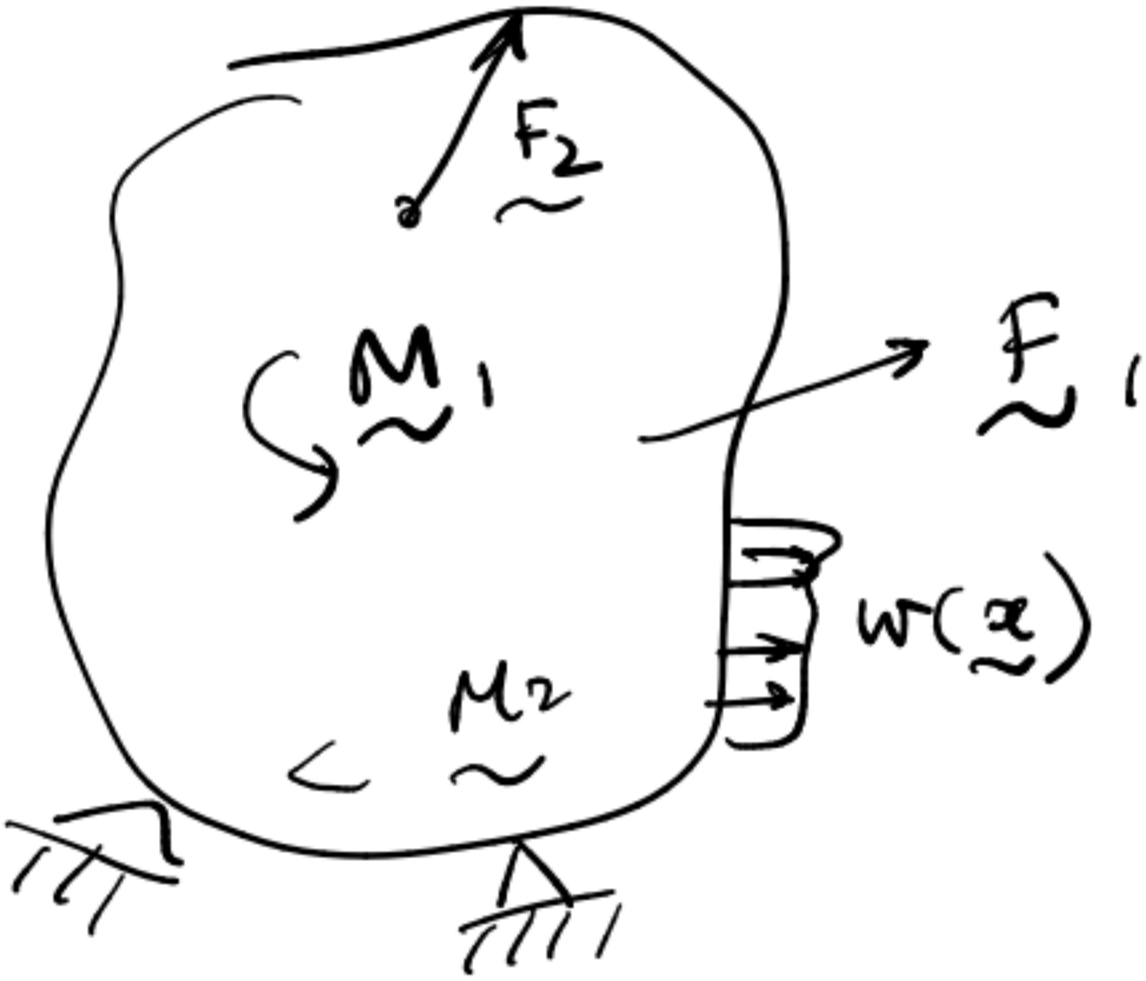
No assumption of
slenderness
⇒ Elastostatic
equation
(eqb).

Kinematic
assumption.

⇏ Elastodynamic
Equs.



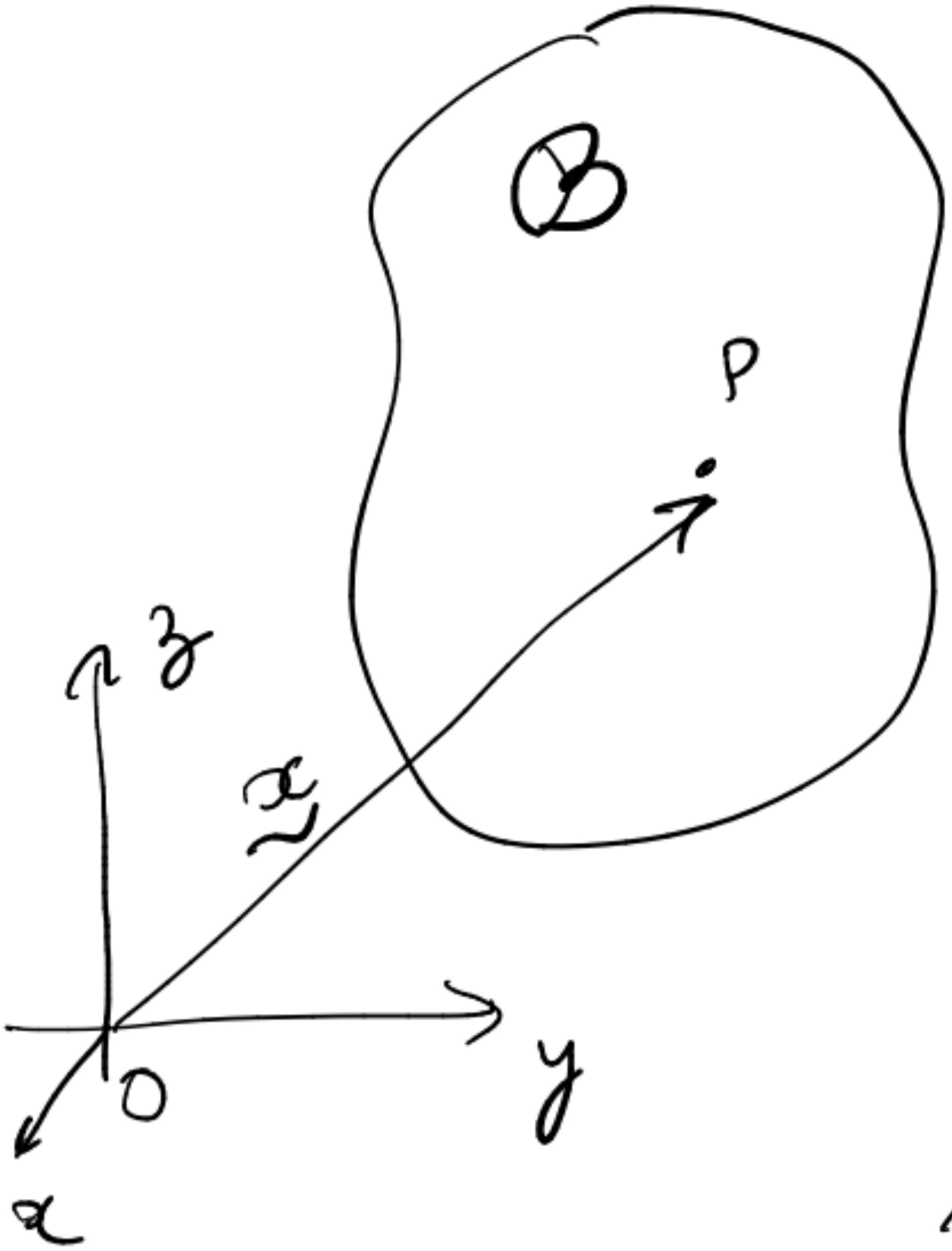
'Fields' Objects 'Bodies.'



Body — is a collection of material points

— It is subject to various forces & moments

— Kept in eqbm. by support



Each point in the body is associated with 'field' quantities:

displacement field

$$\underset{\sim}{u}(\underset{\sim}{x})$$

strain field

$$\underset{\sim}{\varepsilon}(\underset{\sim}{x}).$$

stress field

$$\underset{\sim}{\sigma}(\underset{\sim}{x}).$$

When $\underset{\sim}{u}(\underset{\sim}{x})$, $\underset{\sim}{\varepsilon}(\underset{\sim}{x})$, $\underset{\sim}{\sigma}(\underset{\sim}{x})$ are specified for all $\underset{\sim}{x} \in \mathcal{B}$, we are said to have the 'elasto static state'.

Strength of Materials

- ~~Kinematic assumption~~
- Hooke's law
- Equilibrium Eqns.

Elasticity

"

"

- Strain-Displacement Eqns.
- Compatibility Eqns

Governing.

Equations of elasticity

- Navier Eqns.
- Beltrami- Mitchell eqns.

All elastostatic states satisfy

Fields:

$\underset{\sim}{u}$ — displacement — vector. 1-tensor

$\underset{\sim}{\varepsilon}$ — strain — matrix | 2-tensor

$\underset{\sim}{\sigma}$ — stress — "

We need an efficient / compact way to writes down the eqns governing these field quantities.

§ 1.2 Index notation.

$$u_i \rightarrow \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \equiv \begin{Bmatrix} u_x \\ u_y \\ u_z \end{Bmatrix}$$

Eg:

$\underset{\sim}{u}$ & $\underset{\sim}{v}$ are perpendicular

$$u_1 v_1 + u_2 v_2 + u_3 v_3 = 0.$$

Rule 1 : If an index repeats assume summation from 1 to 3.

$$u_1 v_1 + u_2 v_2 + u_3 v_3 = 0 \quad \text{... original eqn.}$$

$\downarrow$ Rule 1.

$$\longrightarrow u_i v_i = 0$$

$$\text{"} \sum_{i=1}^{3} u_i v_i = 0$$

$$\Rightarrow u_1 v_1 + u_2 v_2 + u_3 v_3 = 0 \text{"}$$

Eg 2 :

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

$$\equiv \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

$$\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = 0.$$

Using Rule 1

$$\varepsilon_{ii} = 0$$

$$" \sum_{i=1}^{3} \varepsilon_{ii} = 0 "$$

Caution: If you find an index repeating more often than twice, the formula is invalid.

Eg 3 $\underset{3\times1}{\{b\}} = \underset{3\times3}{[\varepsilon]} \underset{3\times1}{\{u\}}$ — Matrix vector multiplication

$$\begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{11}u_1 + \varepsilon_{12}u_2 + \varepsilon_{13}u_3 \\ \varepsilon_{21}u_1 + \varepsilon_{22}u_2 + \varepsilon_{23}u_3 \\ \varepsilon_{31}u_1 + \varepsilon_{32}u_2 + \varepsilon_{33}u_3 \end{Bmatrix}$$

$$b_1 = \varepsilon_{11}u_1 + \varepsilon_{12}u_2 + \varepsilon_{13}u_3$$

$$b_2 = \varepsilon_{21}u_1 + \varepsilon_{22}u_2 + \varepsilon_{23}u_3$$

$$b_2 = \varepsilon_{31}u_1 + \varepsilon_{32}u_2 + \varepsilon_{33}u_3.$$

Using Rule 1 we write

$$b_i = \varepsilon_{i\underline{j}}\, u_{\underline{j}}$$

$$\text{"}\quad b_i = \sum_{j=1}^{3} \varepsilon_{ij}\, u_j \quad\text{"}$$

Caution 2: Free (unrepeated) indices should balance on both sides of an eqn.

i is the free index in the above example.