AS5900

Lecture 10.

Displacement gradient, $\nabla \underset{\sim}{u} = u_{i,j}$

Strain, $\underset{\sim}{\varepsilon} = \varepsilon_{ij}$

Rotation, $\underset{\sim}{\omega} = \omega_{ij}$.

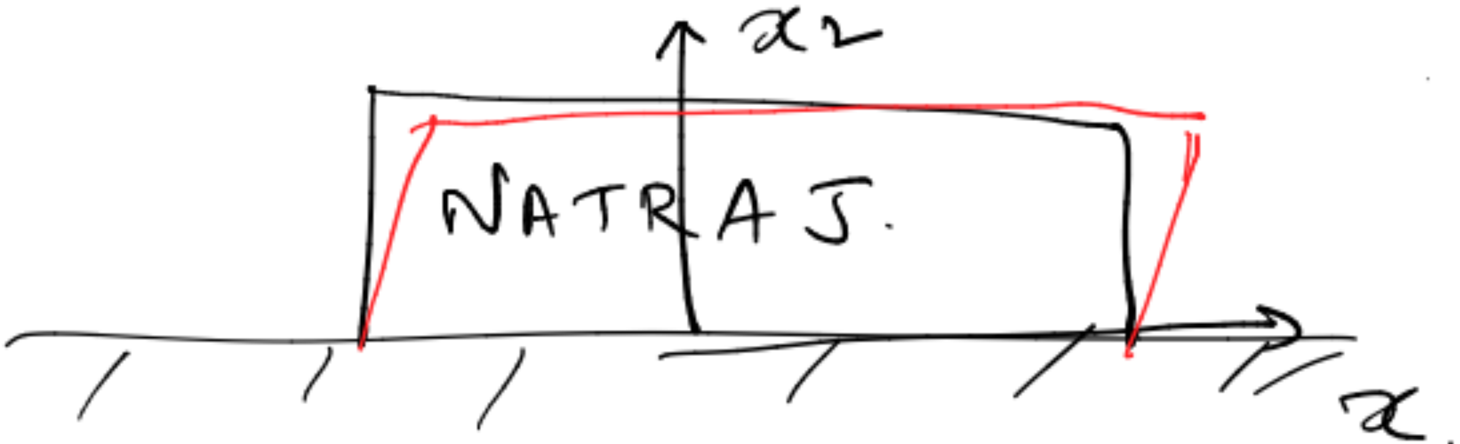
Strain is symmetric $\varepsilon_{ij} = \varepsilon_{ji}$

Rotation is skew, $\omega_{ij} = -\omega_{ji}$

$$\nabla \underset{\sim}{u} = \underset{\sim}{\varepsilon} + \underset{\sim}{\omega}.$$

Eg: Simple shear.

Infinite body under simple shear.



$$\underset{\sim}{u} = x_2 \underset{c}{\cancel{\tan\theta}}\, \underset{\sim}{e_1} = c\, x_2\, \underset{\sim}{e_1}.$$

$$(\nabla \underset{\sim}{u})_{ij} = \begin{pmatrix} 0 & c & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\varepsilon_{ij} = \begin{pmatrix} 0 & c/2 & 0 \\ c/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\omega_{ij} = \begin{pmatrix} 0 & c/2 & 0 \\ -c/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Q: Suppose there is a displ. field for which

$$(\nabla u)_{ij} = \begin{pmatrix} 0 & c/2 & 0 \\ -c/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

What is the corresponding $\underset{\sim}{u}(\underset{\sim}{x})$?

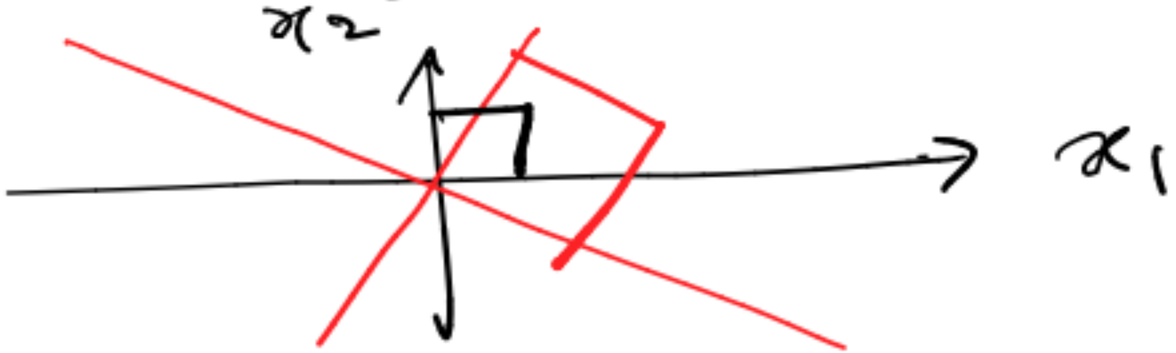
A:

$$\frac{\partial u_1}{\partial x_2} = \frac{c}{2} \Rightarrow u_1(x_1, x_2) = \frac{c}{2} x_2 + \overset{0}{f(x_1)}$$

$$\frac{\partial u_2}{\partial x_1} = -\frac{c}{2} \Rightarrow u_2(x_1, x_2) = \frac{-c}{2} x_1 + \underset{0}{g(x_2)}$$

other $\frac{\partial u_i}{\partial x_j} = 0$.

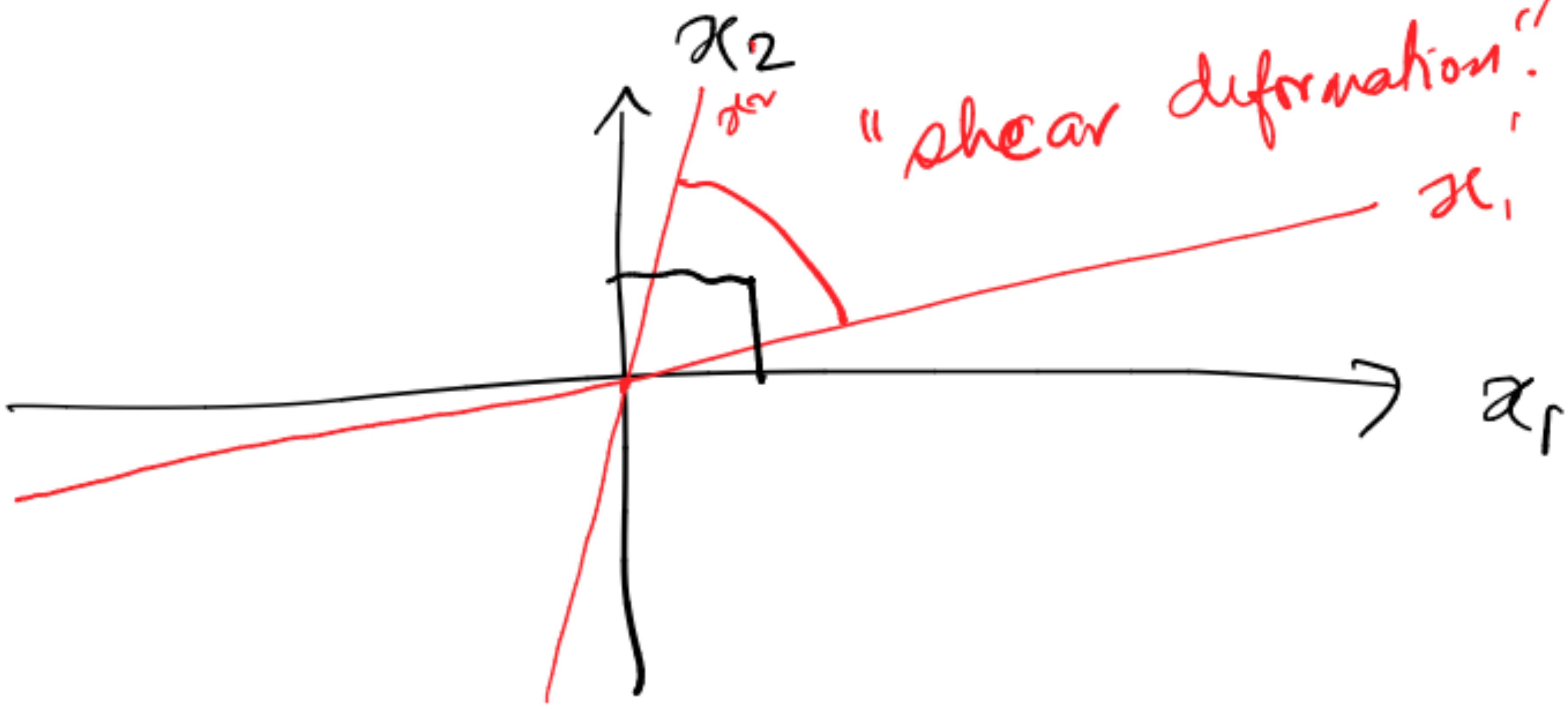
$$\underset{\sim}{u}(\underset{\sim}{x}) = \frac{c}{2} x_2 \underset{\sim}{e_1} - \frac{c}{2} x_1 \underset{\sim}{e_2}.$$



Because $\nabla \underset{\sim}{u} = \underset{\sim}{\omega}$ produces only rotations of the axes & other lines, $\underset{\sim}{\omega}$ is called a rotation.

Next

$$(\nabla u)_{ij} = \begin{pmatrix} 0 & c/2 & 0 \\ c/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\underset{\sim}{u}(\underset{\sim}{x}) = \frac{c}{2} x_2 \underset{\sim}{e_1} + \frac{c}{2} x_1 \underset{\sim}{e_2}.$$

Because angle between the axes changes post $\nabla \underset{\sim}{u}$ application, we have a deformation.

§ 2.2, 2.3 Strain transform

2-tensor

$$\varepsilon'_{ij} = Q_{ip} Q_{jq} \varepsilon_{pq}.$$

apply this formula to 2D to obtain Mohr's circle for strain transformation ← HW.

2.4 principal strains.

3 invariants

$I_1 = \mathrm{tr}\, \varepsilon_{ij} =$ "dilatation".

I_2

$I_3 =$

§ 2.5: Decomposition of strain

$$\varepsilon_{ij} = \underbrace{\tilde{\varepsilon}_{ij}}_{\text{spherical part or hydrostatic part}} + \underbrace{\hat{\varepsilon}_{ij}}_{\text{deviatoric part.}}$$

$$\tilde{\varepsilon}_{ij} = \underbrace{\frac{1}{3}\varepsilon_{kk}}_{\frac{\text{dilation}}{3}}\,\delta_{ij}$$

Captures volume changes.

$$\hat{\varepsilon}_{ij} = \varepsilon_{ij} - \frac{\varepsilon_{kk}}{3}\delta_{ij}$$

distortion of the element

If $(\underset{\sim}{\varepsilon})_{ij} = \begin{pmatrix} 0 & a_2 & 0 \\ a_2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\varepsilon_{ij}' = \hat{\varepsilon}_{ij}'$

$\tilde{\varepsilon}_{ij}' = 0.$

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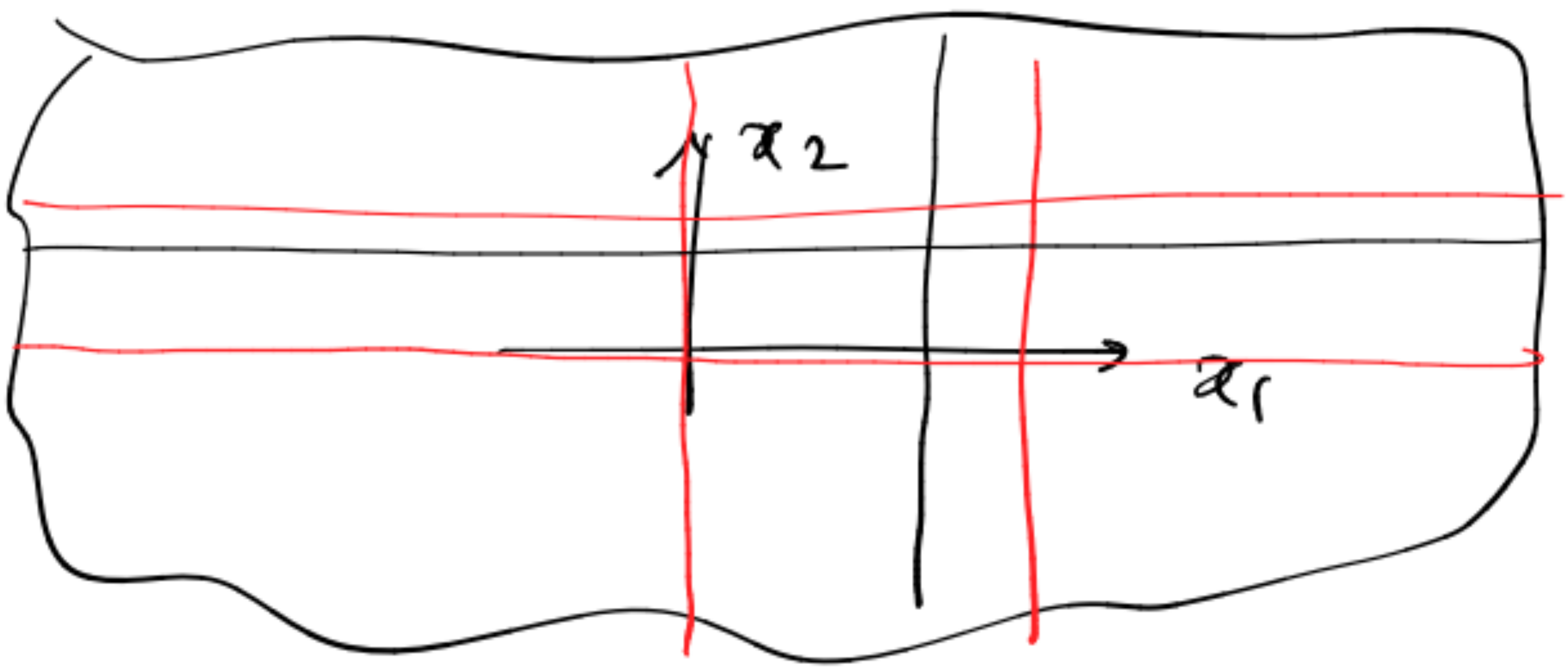
§2.7: St. Venant Equations.

Eg. $\underset{\sim}{u}^{(\alpha)} = c x_1 \underset{\sim}{e_1} + c x_2 \underset{\sim}{e_2}.$

$$(\nabla \underset{\sim}{u})_{ij} = \begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 0 \end{pmatrix} = \varepsilon_{ij}$$

$$\omega_{ij} \equiv 0.$$



Often we must start w/ the strain field ε_{ij} & try to calculate u_i from it.

Q: Can we find the u_i corresponding to arbitrarily prescribed ε_{ij}?

A: No.