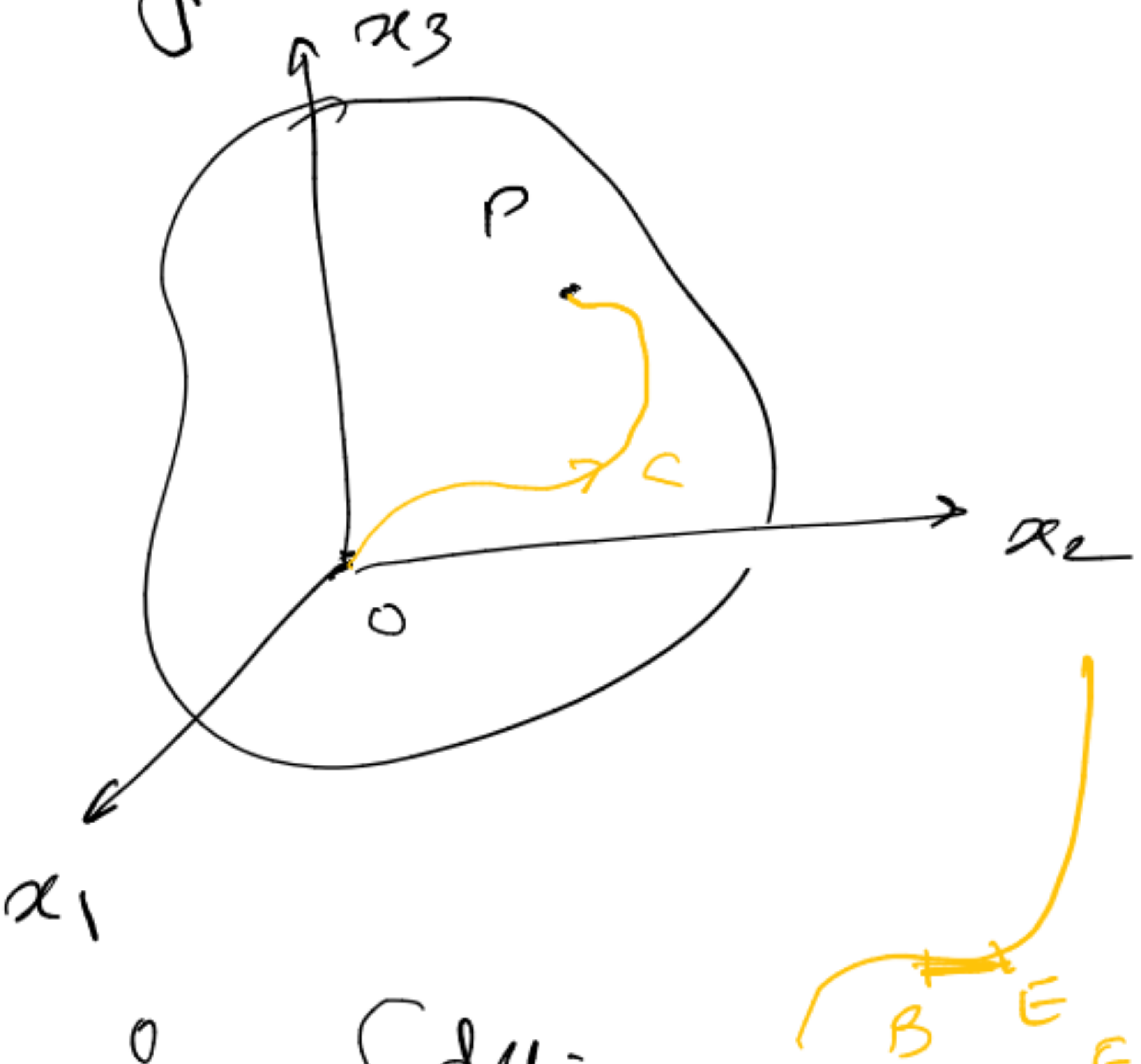
AS5900

Lecture 12

Compatibility Conditions.



$du_i = u_i^E - u_i^B$

$$u_i^P = u_i^O + \int_C du_i$$

$$= u_i^O + \int_C \frac{\partial u_i}{\partial x_j} dx_j$$

↑

$$u_{i,j} = \varepsilon_{ij} + \omega_{ij}$$

$$u_i^P = u_i^0 + \int_C (\varepsilon_{ij} + \underbrace{\omega_{ij}) dx_j}$$

Consider

$$\int_C \underset{u}{\omega_{ij}} \, \underset{dv}{dx_j} = \omega_{ij} x_j \Big|_0^P - \int_C x_j \, d\omega_{ij}$$

$$= \omega_{ij}^P x_j^P - \omega_{ij}^0 \cancelto{0}{x_j^0}$$

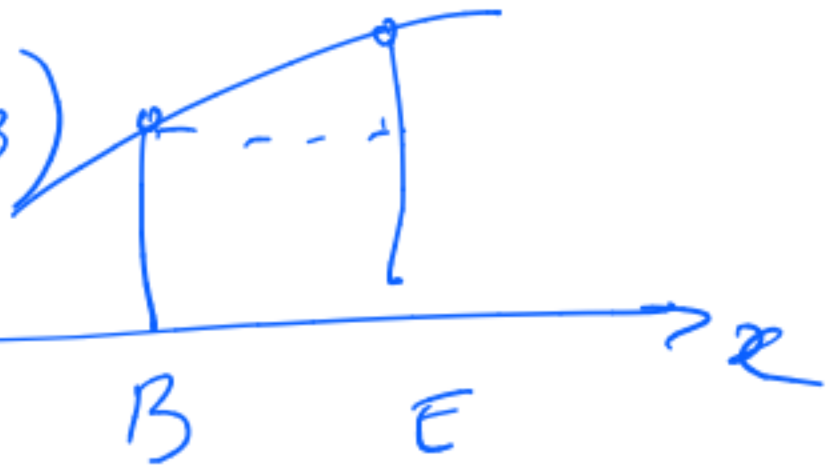
$$- \int_C x_j \omega_{ij,k} \, dx_k$$

$$\sum x_j \underbrace{(\omega_{ij}^E - \omega_{ij}^B)}_{\frac{\partial \omega_{ij}}{\partial x_k} \underbrace{dx_k}}.$$

all pieces

piece centre

$$\omega_E - \omega_B \approx \frac{d\omega}{dx} \cdot (x_E - x_B)$$



$$\int_C \omega_{ij} \, dx_j = \omega_{ij}^P x_j^P - \int_C x_j \omega_{ij,k} \, dx_k.$$

Yesterday:

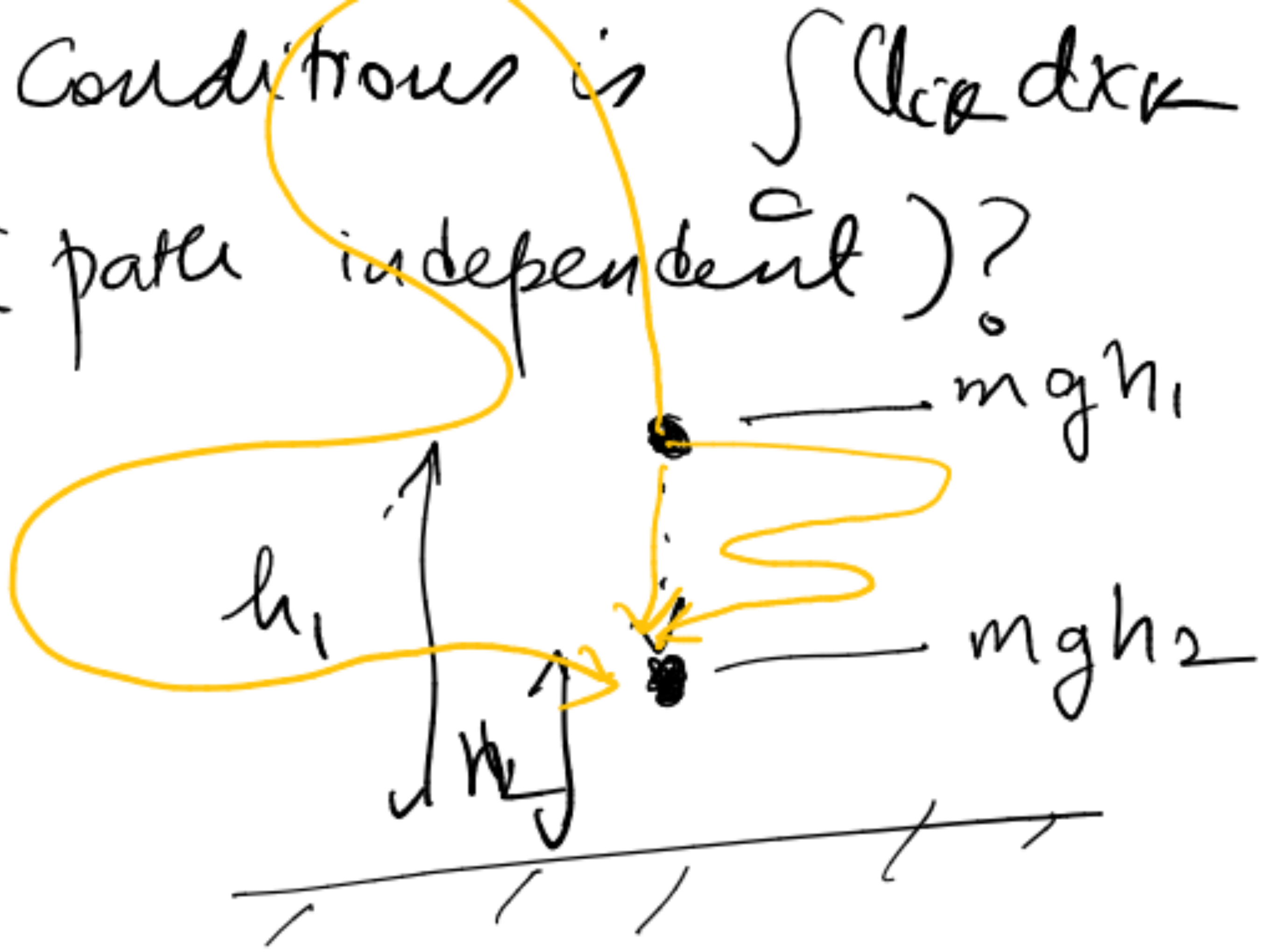
$$\omega_{ij,k} = e_{ik,j} - e_{jk,i}.$$

$$\int_c \omega_{ij}\, dx_j = \quad - \int_c x_j (e_{ik,j} - e_{jk,i})\, dx_k$$

$$u_i^P = u_i^0 + \omega_{ij}^P x_j^P$$
$$+ \int_c \underbrace{(e_{ik} - x_j(e_{ik,j} - e_{jk,i}))}_{U_{ik}} dx_k$$

$$u_i^P = u_i^0 + \omega_{ij}^P x_j^P + \int_c U_{ik}\, dx_k.$$

Unless $\int_c U_{ik}\, dx_k$ is path-independent, we will get different u_i^P for each path that we take under what conditions is $\int_c U_{ik}\, dx_k$ well-posed (path independent)?



$\int_c \underset{\sim}{F} \cdot d\underset{\sim}{r}$ is path-independent

provided $\underset{\sim}{F} = \nabla \phi$

" $\int_c F_{i_k}\, dr_{i_k}$ is " "

provided $F_{i_k} = \phi_{,i_k}$

By the same token, $\int_c U_{ik}\, dx_k$ will be path-independent if

$U_{ik} = \Phi_{i,k}$.

U_{ik} must be of the form $\Phi_{i,k}$.

Is it true } $U_{ik,l} \overset{?}{=} U_{il,k}$

$$\begin{matrix} U_{ik,l} & \overset{?}{=} & U_{il,k} \\ \| & & \| \\ \Phi_{i,kl} & \overset{\checkmark}{=} & \Phi_{i,lk} \end{matrix}$$

If U_{ik} - is of the form $\Phi_{i,k}$

then $U_{ik,l} = U_{il,k}$. $\rightarrow$ Comp. Eqns.

Write this in terms of e_{ij}.

$$\left(e_{ik} - x_j\left(e_{ik,j} - e_{jk,i}\right)\right)_{,l} =$$

$$\left(e_{il} - x_j\left(e_{il,j} - e_{jl,i}\right)\right)_{,k}.$$

$$\Rightarrow e_{ik,l} - \overset{\delta_{jl}}{x_{j,l}}\left(e_{ik,\overset{l}{\cancel{j}}} - e_{\overset{l}{\cancel{j}}k,i}\right) - x_j\left(e_{ik,jl} - e_{jk,il}\right) =$$

$$e_{il,k} - \overset{\delta_{jk}}{x_{j,k}}\left(e_{il,\overset{k}{\cancel{j}}} - e_{\overset{k}{\cancel{j}}l,i}\right) - x_j\left(e_{il,jk} - e_{jl,ik}\right).$$

what is $x_{j,k} = \frac{\partial x_j}{\partial x_k}$

$$\left.\begin{aligned} \frac{\partial x_1}{\partial x_1} &= 1 \\ \frac{\partial x_2}{\partial x_1} &= 0 \\ \frac{\partial x_3}{\partial x_1} &= 0 \end{aligned}\right\} \quad \underset{\underset{\displaystyle x_{j,k}.}{\shortparallel}}{\frac{\partial x_j}{\partial x_k}} = \delta_{jk}.$$

$$x_j \left(e_{ik,jl} - e_{jk,il} - e_{il,jk} + e_{jl,ik} \right) = 0$$

for all x_j.

This implies: $(\;) = 0$

$$e_{ik,jl} - e_{jk,il} - e_{il,jk} + e_{jl,ik} = 0.$$

Compatibility Equation:

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0$$

↳ There are $3^4 = 81$ such eqns.

If a given strain field e_{ij} does not satisfy these 81 conditions, there is no u_i for which $e_{ij} = \frac{u_{i,j} + u_{j,i}}{2}$.

Next: out of these 81, only 6 will survive & out of those 6 only 3 are indep.