AS5900

Lecture 6

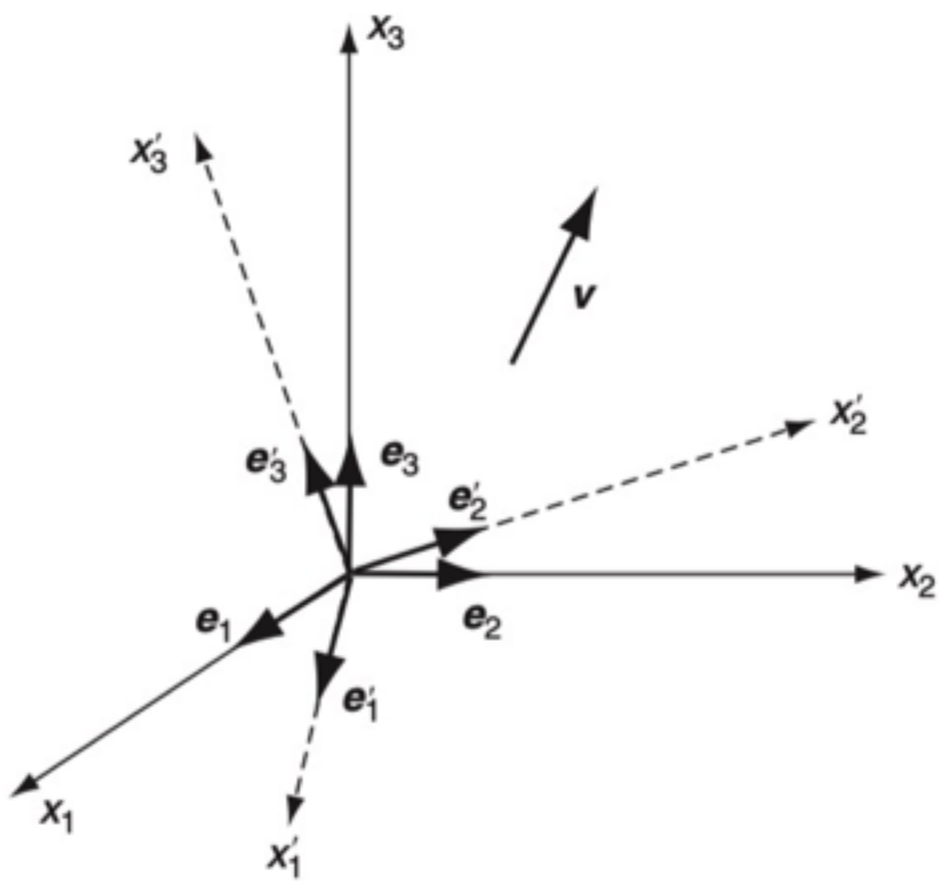


FIGURE 1-1 *Change of Cartesian coordinate frames.*

Obs. 1		Obs 2
xyz		$x'y'z'$
v_i	$\neq$	v_i'
a_{ij}	$\neq$	a_{ij}'
a_{ijk}	$\neq$	a'_{ijk}
a_{ijkl}	$\neq$	a'_{ijkl}

$$a_i' = Q_{ip}\, a_p \quad \cdots\cdots \quad (1)$$

$$a_{ij}' = Q_{ip}\, Q_{jq}\, a_{pq} \quad \cdots \quad (2)$$

Are there invariant quantities that the 2 observers can calculate that are the same.

Suppose the 2 observers decide to calculate the length of $\underset{\sim}{a}$

<u>Observer 1</u>	<u>Observer 2</u>
length of <u>1-tensor</u> $\underset{\sim}{a}$	" " "
Obs. 1 has components (a_1, a_2, a_3) of 1-tensor $\underset{\sim}{a}$.	Obs 2 " " (a_1', a_2', a_3') of 1-tensor $\underset{\sim}{a}$.

$\|\underset{\sim}{a}\| =$ "length of $\underset{\sim}{a}$" $= \sqrt{a_i a_i}$

$\|\underset{\sim}{a}\| =$ "length of $\underset{\sim}{a}$" $= \sqrt{a_i' a_i'}$

Are these 2 equal.

$$\sqrt{a_i' a_i'} \overset{(1)}{=} \sqrt{Q_{ip} a_p Q_{iq} a_q}$$

$$= \sqrt{Q_{ip} Q_{iq} a_p a_q}$$

We know: $Q Q^T = I$

$$Q_{ip} Q^T_{pj} = \delta_{ij}$$

$$\Rightarrow Q_{ip} Q_{jp} = \delta_{ij}$$

$Q^T Q = I$

$Q^T_{ip} Q_{pj} = \delta_{ij}$

$Q_{pi} Q_{pj} = \delta_{ij}$

$$= \sqrt{\delta_{pq} a_p a_q}$$

$$= \sqrt{a_q a_q} = \sqrt{a_i a_i}$$

$i \leftarrow p$, $p \leftarrow i$

$q \leftarrow j$

$Q_{ip} Q_{iq} = \delta_{pq}$

So it is true that

$$\|\underset{\sim}{a}\| = \sqrt{a_i a_i} = \sqrt{a_i' a_i'}.$$

Are there invariant quantities associated with components of 2-tensor that xyz & $x'y'z$ can calculate, which are the same?

Yes.

Obs. 1

a_{ij}

$a_{ij} n_j = \lambda n_i$ (3)

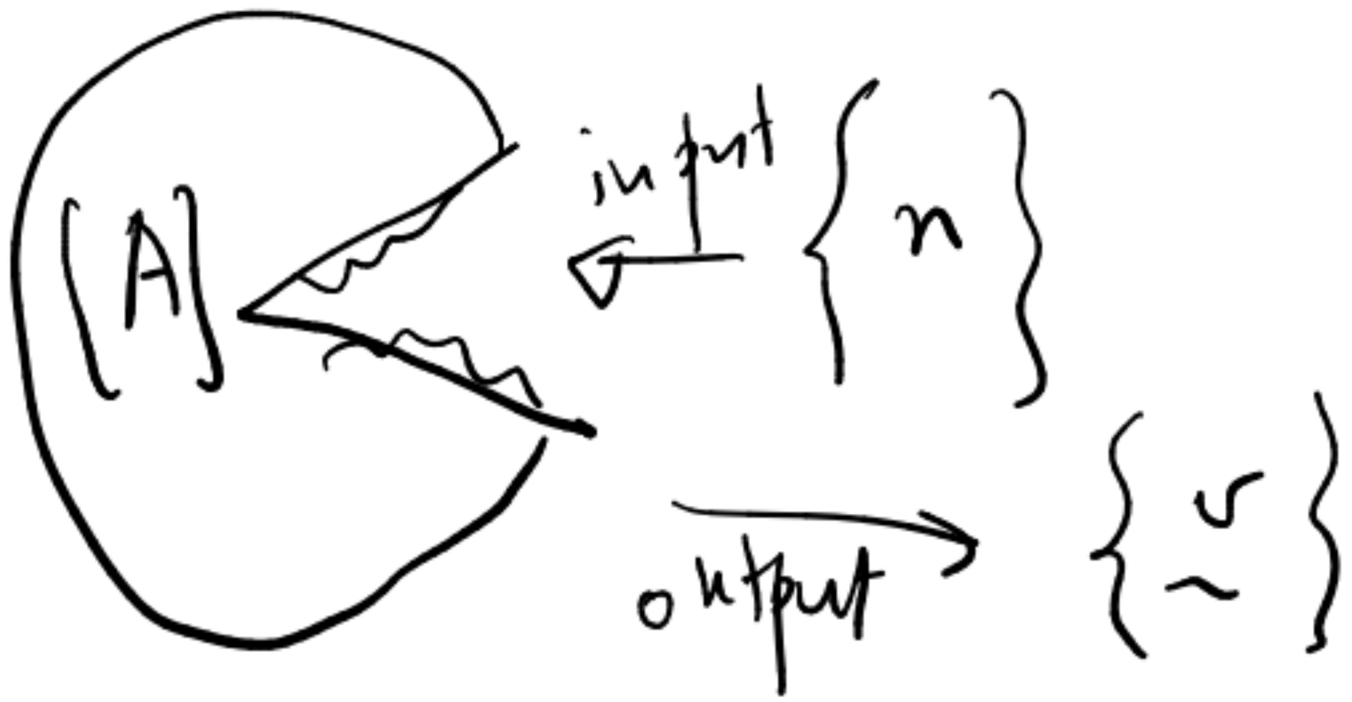
Obs. 2

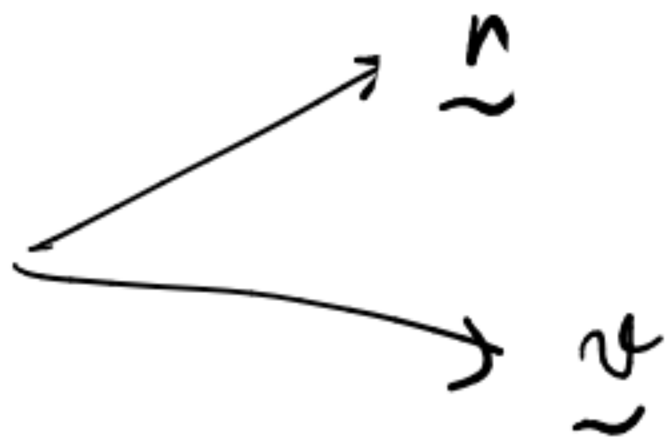
$a_{ij}' = Q_{ip} Q_{jq} a_{pq}$

$a_{ij}' n_j' = \lambda n_i'$ (4)

Q: Will both Obs 1 & obs 2 calculate the same 1-tensor $\underset{\sim}{n}$ from this calculation?

$[A]\{u\}$ Explanation of matrix product.





$a_{ij} u_j = \lambda u_i \Rightarrow$ output of feeding u_j to a_{ij} is $\parallel$el to input u_j.

Yes... if n_j & n_j' are related

thru' eqn. ①, i.e,

$$n_i' = Q_{ip} n_p \quad \text{—— ①}$$

& λ's are the same

prove that the above holds.

We should show that if (n_j', λ) satisfy eqn. ④, (n_j, λ) satisfies eqn. ③ :

Start from ④. Let n_j', λ satisfy ④

$$a_{ij}' n_j' = \lambda n_i'$$

② ⇓

$$\Rightarrow Q_{ip} Q_{jq} a_{pq} Q_{jk} \underbrace{n_k}_{= n_q} =$$

δ_{qk}

$$\lambda Q_{ip} n_p$$

$$Q_{ip}\, a_{pq}\, n_q = \lambda Q_{ip}\, n_p .$$

Direct cancellation of Q_{ip} is not allowed.

Multiply both sides with Q_{ik}.

$$\underbrace{\underbrace{Q_{ik}\, Q_{ip}}_{\delta_{kp}}\, a_{pq}\, n_q}_{a_{kq} n_q} = \underbrace{\lambda \underbrace{Q_{ik}\, Q_{ip}}_{\delta_{kp}}\, n_p}_{\lambda n_k}$$

$$\boxed{a_{kq}\, n_q = \lambda n_k} \to \text{Eq. } 3$$

$$Q_{ip}\, Q_{iq} = \delta_{pq}$$

$$Q_{ip}\, Q_{jp} = \delta_{ij}$$

1- tensors $\underset{\sim}{n}$ whose components in xyz & $x'y'z'$ system are n_i & n_i' & which satisfy

$$a_{ij} n_j = \lambda n_i \quad \Big| \text{or} \Big| \quad a'_{ij} n'_j = \lambda n'_i$$

are called the principal directions of the 2- tensor $\underset{\sim}{a}$.

The principal directions $\underset{\sim}{n}$ are objective 1- tensors. But to compute them, one MUST fix a coord. system & work with the subjective matrix a_{ij}, or a'_{ij}.