AS5900

Lecture 8

Yesterday:

Eg 1:

$$a_{ij} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$

$\lambda = 5, 2, -5 \rightarrow$ distinct.

3 $\perp^r$ eigenvectors.

Eg 2:

$$a_{ij} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & -3 \end{bmatrix}$$

Finding eigenvalues.

$$\det\left(a_{ij} - \lambda \delta_{ij}\right) = 0$$

$$\begin{vmatrix} 5-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 4 & -3-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)\left\{(3-\lambda)(-3-\lambda)-16\right\} = 0$$

$$\Rightarrow (5-\lambda)\left(\lambda^2 - 25\right) = 0$$

$$\Rightarrow (5-\lambda)(\lambda+5)(\lambda-5) = 0$$

$$\Rightarrow \lambda = -5, \underbrace{5, 5}_{2}$$ eigenvalues coincide.

Eigenvector for $\lambda = -5$:

$$(a_{ij} - \underset{\substack{\uparrow \\ -5}}{\lambda}\delta_{ij})\, n_j^{(1)} = 0 \longrightarrow$$

$$\begin{pmatrix} 10 & 0 & 0 \\ 0 & 8 & 4 \\ 0 & 4 & 2 \end{pmatrix} \begin{Bmatrix} n_1^{(1)} \\ n_2^{(1)} \\ n_3^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$n_1^{(1)} = 0$ ... 1st eqn.

2nd & 3rd eqns are the same.

$$2n_2^{(1)} + n_3^{(1)} = 0 \Rightarrow n_2^{(1)} = -\frac{1}{2} n_3^{(1)}.$$

$$\begin{Bmatrix} n_1^{(1)} \\ n_2^{(1)} \\ n_3^{(1)} \end{Bmatrix} = \alpha \begin{Bmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{Bmatrix} \quad \ldots \text{ corresp to } \lambda = -5.$$

$\lambda = 5 \ldots$ Consider

$$(a_{ij} - \lambda \delta_{ij})\, n_j^{(2)} = 0$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2 & 4 \\ 0 & 4 & -8 \end{pmatrix} \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

1st equ — useless.

2nd & 3rd equs — same.

$$-\cancel{2}\, n_2 + \cancel{4}^{\,2}\, n_3 = 0.$$

$$\Rightarrow \boxed{n_2 = 2 n_3.}$$

$$\begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix} = \begin{Bmatrix} \alpha' \\ 2\beta' \\ \beta' \end{Bmatrix} \text{ for any real } \alpha \,\&\, \beta.$$

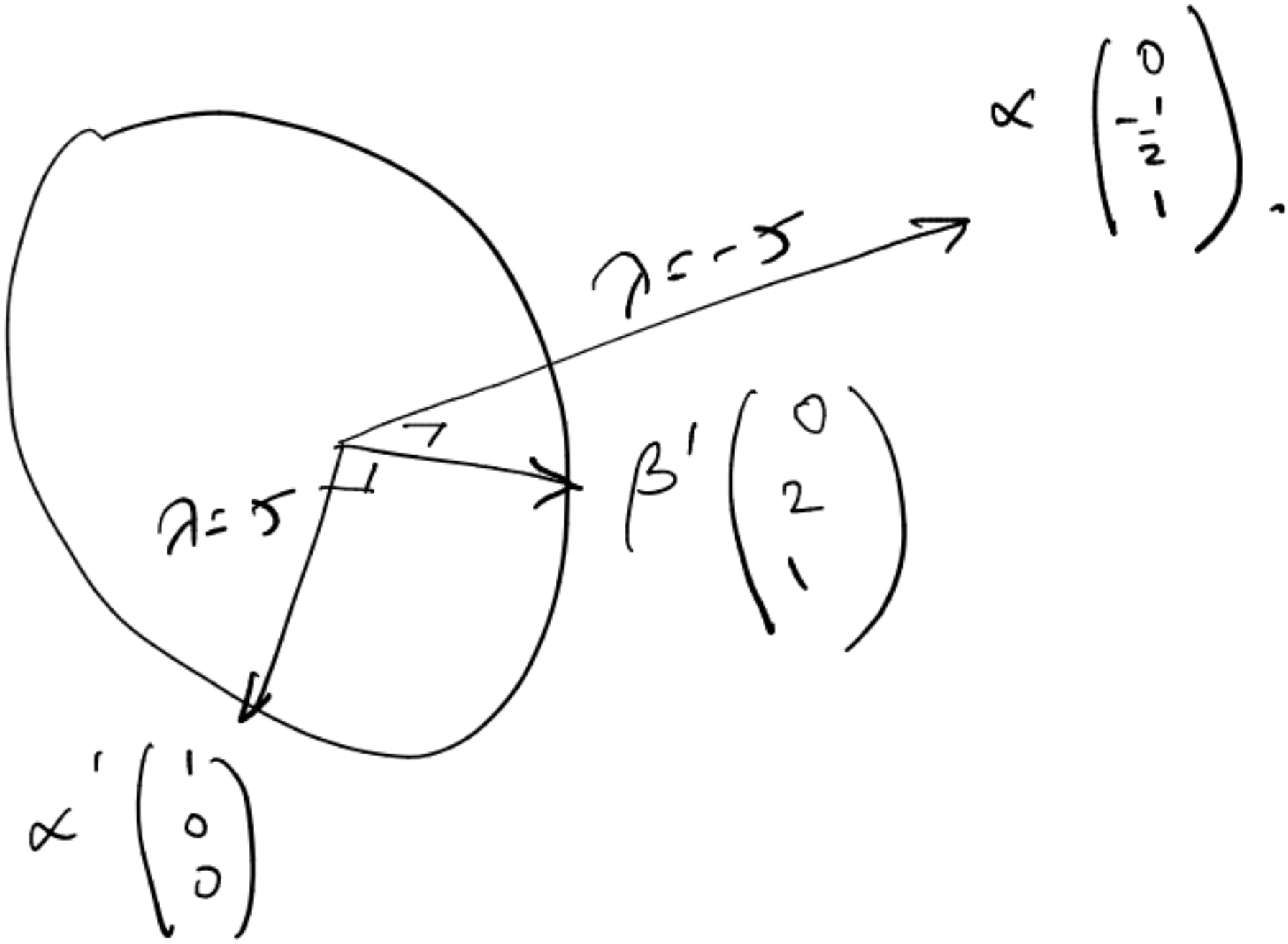
$$= \alpha' \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} + \beta' \begin{Bmatrix} 0 \\ 2 \\ 1 \end{Bmatrix}$$

<u>Property:</u>

$$\begin{Bmatrix} 0 \\ -\frac{\alpha}{2} \\ 1\alpha \end{Bmatrix} \cdot \begin{Bmatrix} \alpha' \\ 2\beta' \\ \beta' \end{Bmatrix} = -\frac{\alpha}{2} \cdot 2\beta' + \alpha\beta' = 0.$$

e.v. for $\lambda = -5$ e.v. for $\lambda = 5$.

" The 2 eigenvectors corresp to $\lambda = 5$ (repeated eigenvalue) are $\perp^r$ to the eigenvector corresp to $\lambda = -5$ (non-repeating eigenvalue).



1.7: Vector, matrix, tensor algebra.

$$\underset{\sim}{a} \cdot \underset{\sim}{b} = a_i b_i$$

$$\| \underset{\sim}{a} \| = \sqrt{a_i a_i}$$

$$(\underset{\sim}{a} \times \underset{\sim}{b})_i = \varepsilon_{ijk} a_j b_k$$

$$\underset{\sim}{a} \times \underset{\sim}{b} = \varepsilon_{ijk} a_j b_k \underset{\sim}{e_i}.$$

$$([A]\{a\})_i = A_{ij}\, a_j$$

$$A^T_{ij} = A_{ji}$$

$$(AB)_{ij} = A_{ik}\, B_{kj}$$

$$\underset{\sim}{A}\,\underset{\sim}{B} = A_{ik} B_{kj}\, \underset{\sim}{e_i} \otimes \underset{\sim}{e_j}.$$

$$tr(A) = A_{ii}$$

1.8 Calculus of Cartesian tensors.

$f(\underset{\sim}{x})$ scalar field

$$\nabla f = f_{,i}\, \underset{\sim}{e_i}.$$

$$\nabla^2 f = \nabla \cdot (\nabla f)$$

$$= \nabla \cdot (f_{,i}\, \underset{\sim}{e_i})$$

$$= f_{,ii}.$$

Prove the identity:

$$\nabla \cdot (\phi \underset{\sim}{u}) = \nabla \phi \cdot \underset{\sim}{u} + \phi (\nabla \cdot \underset{\sim}{u}).$$

ϕ: scalar field

$\underset{\sim}{u}$: vector field

<u>LHS:</u> $\nabla \cdot (\phi \underset{\sim}{u}) = (\phi u_i)_{,i}$

chain rule $= \phi_{,i} u_i + \phi u_{i,i}$.

$$= (\nabla \phi)_i u_i + \phi \nabla \cdot \underset{\sim}{u}$$

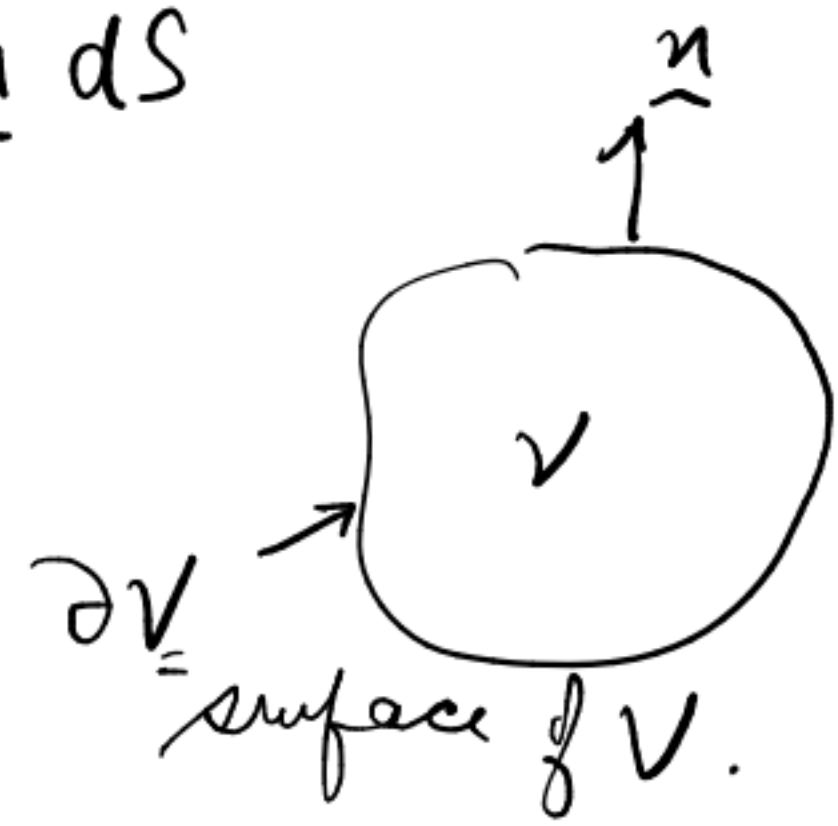
$$= \nabla \phi \cdot \underset{\sim}{u} + \phi \nabla \cdot \underset{\sim}{u}.$$

<u>H/W:</u> prove other identities given in eq. 1.8.5 of the book. p. 19.

Divergence theorem:

Let $\underset{\sim}{u}$ be a vector field.

$$\iiint_V \nabla \cdot \underset{\sim}{u} \, dV = \iint_{\partial V} \underset{\sim}{u} \cdot \underset{\sim}{n} \, dS$$



Stokes' theorem:

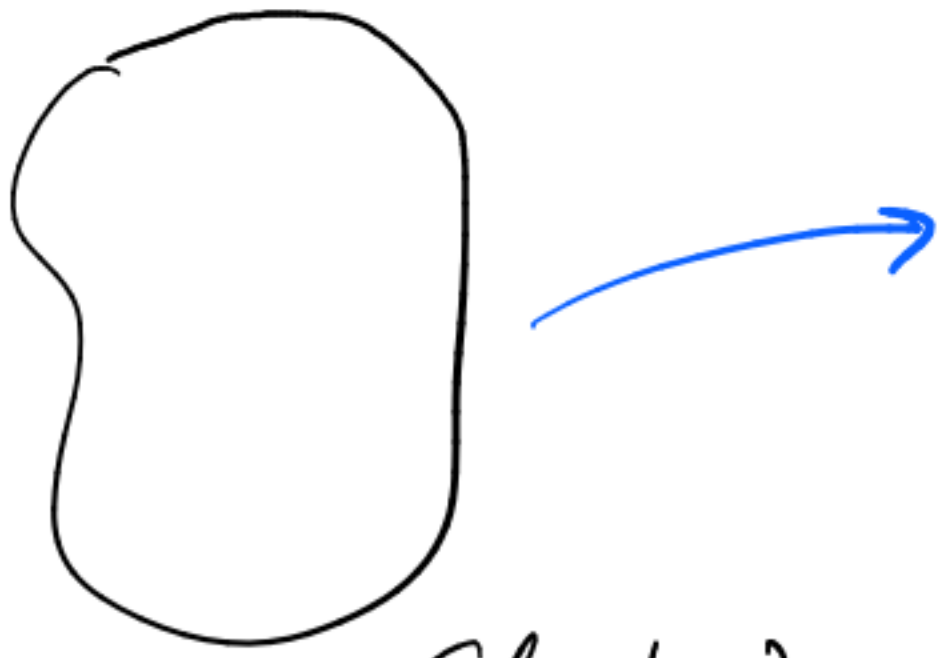
$$\int_C \underset{\sim}{u} \cdot d\underset{\sim}{r} = \iint_S (\nabla \times \underset{\sim}{u}) \cdot \underset{\sim}{n} \, dS$$

~ Chap 1 ~

HW: Ex. 1.1 — 1.17.

Chap 2:

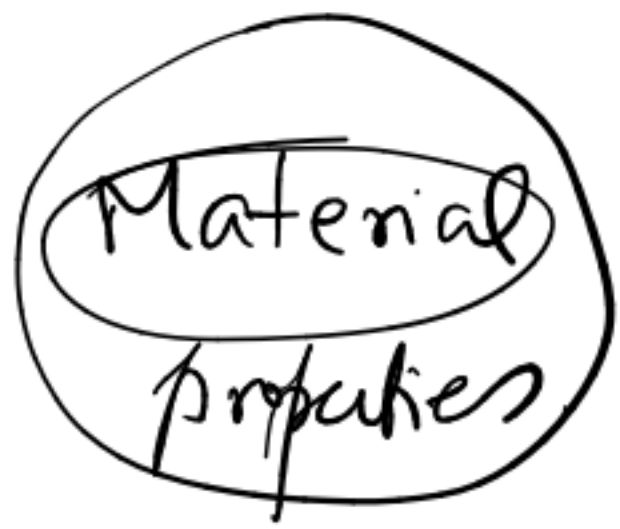
Kinematics — Motions & Deformations.



Chap 3:

Kinetics — forces.

Chap 4:

Kinematics 

Kinetics.

Chap 5

Boundary value problems of elasticity.

Chap 6:

Energy methods of solution.

Chap 8:

Analytical solns to important 2D problems.